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A KEY

TO

BONNYCASTLE'S

INTRODUCTION TO MENSURATION

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A New Edition.

LONDON:

PRINTED FOR THOMAS TEGG, 73, CHEAPSIDE

1840.

GLASGOW:
PRINTED BY GEORGE BROOKMAN & CO.

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The subjects handled in this volume are doubtless of the utmos value to every practical mechanic, and indeed to every perso who forms a due estimate of the importance of science, or wh duly feels the pleasure arising from the acquirement of know ledge.

The editor thinks it right to state, that he has inserted considerable number of original problems, and worked thei all out at full length; a method which is highly convenient t tutors, and which affords great assistance to those who undei take the meritorious labour of self-instruction.

It is recommended that some of the problems in this wor be omitted by the junior pupils; especially those problems whic relate to the surfaces and solidities of elliptic and circul spindles: for instance, Problem XXIV. to Problem XXVII inclusive; also Problems XXXII., XXXIII. Solids of thes kinds are seldom met with; the solutions of Problems relatin to them are very intricate, and the rules extremely burdensom to the memory.

WAKEFIELD, 1835.

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KEY

TO

BONNYCASTLE'S MENSURATION.

MENSURATION OF SUPERFICIES.

PROBLEM I.

RULE I.

Ex. 5. What is the area of a square whose side is 17·625 chains.

```
        17·625
        17·625
        ------
         88125
        35250
      105750
      123375
      17625
     -------
  10)310·640625  sq. chains.
     ---------
     31·0640625 sq. acres.
              4
     ----------
       ·2562500
             40
     ----------
     10·2500000
     ----------
```

Ans. 31 *sq. acres,* 0 roods, 10¼ poles

6. What is the area of a square whose side is 35·25 ch

$$
\begin{array}{r}
35.25 \\
35{\cdot}25 \\
\hline
17625 \\
7050 \\
17625 \\
10575 \\
\hline
10)124{\cdot}25625 \text{ sq. chains.} \\
\hline
124{\cdot}25625 \text{ sq. acres.} \\
4 \\
\hline
1{\cdot}02500 \text{ roods.} \\
40 \\
\hline
1{\cdot}00000 \\
\hline
\end{array}
$$

Ans. 124 *sq. acres*, 1 *rood*, 1 *pc*

7. What is the area of a rectangle whose length is 14
6 inches, and breadth 4 feet 9 inches?

feet.	*inches.*	
14	6	
4	9	
58	0	
10	10	6
68	10	6

Ans. 68 *feet*, 10 parts, 6 inc

By Decimals.

feet.	*inches.*		
14	6	=	14·5 feet.
4	9	=	4·75 feet.

```
    725
  1015
  580
  ------
  68·875
      12
  ------
  10·500
      12
  ------
   6·000
```

Ans. 68 *feet,* 10 *parts,* 6 *inches,* as before.

.equired the area of a rhombus, the length of whose side .4 feet, and height 9·16 feet?

```
    12·24
     9·16
   ------
     7344
    1224
  11016
  --------
  112·1184
        12
  --------
     14208
        12
  --------
    5·0496
```

Ans. 112 *feet,* 1 part, 5 inches

9. Required the area of a rhomboid whose length is chains, and breadth 4·28 chains.

```
        10·51
         4·28
        ------
         8408
        2102
       4204
       ------
   10)44·9828  sq. chains.
       ------
        4·49828 acres.
              4
       ------
        1·99312 roods.
             40
       ------
       39·72480 perches.
       ------
```

Ans. 4 *acres*, 1 *rood*, 39 *p*

10. What is the area of a rhomboid whose length is 9 inches, and height 3 feet 6 inches?

By Duodecimals.

feet.	*inches.*	
7	9	
3	6	
23	3	
3	10	6
27	1	6

Ans. 27 *feet*, 1 *part*,

By Decimals.

$$\begin{array}{r} 7{\cdot}75 \\ 3{\cdot}5 \\ \hline 3875 \\ 2325 \\ \hline 27{\cdot}125 \\ 12 \\ \hline 1{\cdot}500 \\ 12 \\ \hline 6{\cdot}000 \\ \hline \end{array}$$

Ans. 27 *feet,* 1 *part,* 6 *inches,* as before.

11. To find the area of a rectangular board, whose length is $12\frac{1}{2}$ feet, and breadth 9 inches?

feet.	*inches.*	
12	6	
	9	
9	4	6

Ans. 9 *feet,* 4 *parts,* 6 *inches.*

By Fractions.

$12\frac{1}{2} = \frac{25}{2}$ feet.

9 inches, $= \frac{3}{4}$ feet.

$\therefore$ area required $= \frac{25}{2} \times \frac{3}{4} = \frac{75}{8} = 9\frac{3}{8}$ sq. feet, as before.

12. What is the area of a rectangular court yard, whose length is $46\frac{1}{3}$ feet, and breadth $15\frac{1}{2}$ feet?

$46\frac{1}{3} = \frac{139}{3}$ feet.

$15\frac{1}{2} = \frac{31}{2}$ feet.

$\therefore$ *area* $= \frac{139}{3} \times \frac{31}{2} = \frac{4309}{6} = 718$ feet, 2 parts

13. Determine the number of square yards in a rho whose length is 37 feet, and breadth 5 feet 3 inches.

feet.	*inches.*	
37	0	
5	3	
185	0	
9	3	
9)194	3	
21	5	3

Ans. 21 *sq. yards,* 5 *feet,* 3 *parts;* or 21 *sq. yards,* *feet,* 36 *sq. inches.*

RULE II.

Ex. 2. Required the area of a rhombus, a side of w 12·5 chains, and the acute angle contained by two ac sides 30°.

$$\begin{array}{r} 12{\cdot}5 \\ 12{\cdot}5 \\ \hline 625 \\ 1500 \\ \hline 2)156{\cdot}25 \\ \hline \end{array}$$

Area = 78.125 sq. chains.

Note.—Since the nat. sine 30° = ½ when radius = unity; therel the product of two adjacent sides is equal to the continued produc two sides and the fraction ½.

3. *Find the area of a rhomboid, two of whose adjacen are 25·35 and 10·4* chains, and their included angle 60°

25·35
10·4

10140
2535

263·640
·866 = nat. sin. 60° to rad. 1.

1581840
158184
210912

10)228·312240 sq. chains.

22·831224 sq. acres.
4

3·324896 roods.
40

·12·995840

Ans. 22 *acres,* 3 *roods,* 13 *poles, nearly.*

Find the same as in the last problem when the included is 45°.

25·35 × 10·4 = 263·64
·7071 = nat. sin, 45°.

26364
184548
184548

10)186·419844 sq. chains.

18·6419844 sq. acres.
4

2·5679376 sq. roods.
40

22·7175040 sq. poles.

Ans. 18 *acres,* 2 *roods,* 22 *pole*

PROBLEM II.

Ex. 2. What is the area of a triangle whose base is 11
10 inches, and perpendicular altitude 9 feet 2 inches?

By Duodecimals.

feet.	*inches.*	
11	10	
9	2	
106	6	
1	11	8
2)108	5	8
54	2	10

Ans. 54 *feet*, 2 *parts*, 10 *in*

By Decimals.

11 feet 10 inches = 11·8333
9 feet 2 inches = 9·1666

```
          709998
         709998
        709998
       118333
     1064997
    -----------
    2)108·47112778
    -----------
      54·23556389
               12
    -----------
       2·82676668
    -----------
       9·92120016
```

Ans 54 *feet*, 2 *parts*, 10 inches, very

What is the area of a triangle whose base is 10 feet 5 ;, and height 7 feet 4 inches?

feet.	*inches.*	
10	5	
7	4	
72	11	
3	5	8
2)76	4	8
38	2	4

Ans. 38 *sq. feet,* 2 *parts,* 4 *inches.*

What is the area of a triangle whose base is 15·75 feet, ight 14·5 feet?

```
     15·75
     14·5
     -----
     7875
    6300
   1575
   ------
 2)228·375
  --------
  114·1875
        12
  --------
    2·2500
        12
    ------
    3·0000
```

Ans. 144 *feet,* 2 *parts,* 3 *inches.*

'*equired the area* of a triangle whose base is 12·6 chains, *'ude* 6.4 *chains.*

chains.

$$\begin{array}{r} 12{\cdot}6 \\ 6{\cdot}4 \\ \hline 504 \\ 756 \\ \hline 2)80{\cdot}64 \\ \hline \end{array}$$

Ans. = 40.32 sq. chains.

6. There is a triangle whose base is $18\frac{1}{2}$ yards in le
and its perpendicular altitude $15\frac{1}{4}$ yards; required its are

$18\frac{1}{2} = 18{\cdot}5$ yards.
$15\frac{1}{4} = 15{\cdot}25$ yards.

$$\begin{array}{r} 7625 \\ 12200 \\ 1525 \\ \hline 2)282{\cdot}125 \\ \hline \end{array}$$

Ans. = $141{\cdot}0625$ sq. yards.

7. Find the area of a triangle, two of whose sides are 3(
40 yards, and their included angle 30°

$$\begin{array}{r} 30 \\ 40 \\ \hline 1200 \\ {\cdot}5 \\ \hline 2)600{\cdot}0 \\ \hline \end{array}$$

$\cdot 5$ = nat. sin. 30°.

Area = 300 sq. yards. *Ans.*

'nd the area of a triangle, two of whose sides
ards, and the included angle 60°.

```
          45
          37
        ----
         315
        135
        ----
        1665
        ·866  = nat. sine 60°.
        ----
        9990
       9990
      13320
      ------
   2)1441·890
      ------
Area =  720·945 sq. yards.   Ans.
      ------
```

. Find the same as in the last problem, when the included .e is 45°.

```
          45
          37
        ----
         315
        135
        ----
        1665
       ·7071  = nat. sine 45°.
        ----
        1665
      11655
    11655
    -------
  2)1177·3215
    -------
Area =  588·66075 sq. yards.   Ans.
    -------
```

PROBLEM III.

'x. 3. *Required the area of a* right-angled triangle, who *thenuse is 50,* and the other two sides 30 and 40.

In this example there is no necessity for employing the rule usually adopted when three sides are given to find the area, because the triangle is a right-angled one; consequently its area $= \text{base} \times \frac{\text{perp.}}{2} = \frac{40 \times 30}{2} = 600$, the answer required.

4. Determine the area of an equilateral triangle one of whose sides is 25.

$$\text{Perimeter} = 75$$
$$\tfrac{1}{2}\ \text{Perimeter} = 37{\cdot}5$$
$$\text{and}\ \tfrac{1}{2}\ \text{Perimeter} - \text{one side} = 12{\cdot}5$$

consequently the area $= (37{\cdot}5 \times 12{\cdot}5 \times 12{\cdot}5 \times 12{\cdot}5)^{\frac{1}{2}} =$ $\sqrt{37{\cdot}5 \times (12{\cdot}5)^3} = \sqrt{73242{\cdot}1875} = 270{\cdot}632$, the answer required.

5. Required the area of an isosceles triangle whose base is 20, and each of its equal sides 15.

	20	1st remainder = 5
	15	2d do. = 10
	15	3d do. = 10
perimeter =	50	
	2)—	
½ perimeter =	25	

$\therefore$ area $= \sqrt{25 \times 5 \times 10 \times 10} = \sqrt{12500} = 111{\cdot}803$, as required.

6. Required the area of a triangle whose three sides are 20, 30, and 40 chains.

	20	½ Perimeter. = 45
	30	1st Remainder. = 25
	40	2d do. = 15
	—	3d do. = 5
Perimeter =	90	

$= \sqrt{45 \times 25 \times 15 \times 5} = 75\sqrt{15} = 3{\cdot}873 \times 75$
5 sq. chains = 39.0475 sq. acres = 29 acres, ...
oles, very nearly.

7. How many acres are there in a triangle whose three sid are 49, 50·25, and 25·69 chains?

50·25	$\frac{1}{2}$ perimeter	= 62·47
49·	1st remainder	= 12·22
25·69	2d do.	= 13·47
	3d do.	= 36·78

perimeter = 124·94

$\therefore$ area $= (62{\cdot}47 \times 12{\cdot}22 \times 13{\cdot}47 \times 36{\cdot}78)^{\frac{1}{2}} =$ $\sqrt{378200{\cdot}44235844} = 614{\cdot}98$ sq. chains = 61·498 acres, th answer required.

8. Determine the number of acres contained in a triangul field whose sides are 320, 180, and 160 yards.

320	$\frac{1}{2}$ perimeter	= 330
180	1st remainder	= 10
160	2d do.	= 150
	3d do.	= 170

perimeter = 660

$\therefore$ area $= \sqrt{330 \times 10 \times 150 \times 170} = 100\sqrt{33 \times 15 \times 17} =$ 9173·33 sq. yards. Now 4840 sq. yards = 1 acre.
$\therefore$ 9173·33 divided by 4840 = 1·895 acres = 1 acre, 3 rood 23 poles.

9. There is a triangular cornfield whose three sides a 150, 200, and 250 yards respectively; determine the numb of acres contained in the field, and the expense of reaping th corn at 9*s*. 6*d*. per acre.

250	$\frac{1}{2}$ perimeter	= 300
200	1st remainder	= 50
150	2d do.	= 100
	3d do.	= 150

perimeter = 600

$\therefore$ area $= \sqrt{300 \times 50 \times 100 \times 150} = 100\sqrt{30 \times 5 \times 10 \times 15}$ $1000\sqrt{15 \times 15} = 15000$ sq. yards; which being divide 4840 gives 3·09917 acres.

```
          3·09917 acres.
                9
          -------
          27·89253
6d. is ½   1·54958
          -------
          29·44211
                12
          -------
           5·30532
                 4
          -------
           1·22128
          -------
```

Expense of reaping $= 1l.\ 9s.\ 5\frac{1}{4}d.$ nea

PROBLEM IV.

Ex. 3. The base of a right-angled triangle is 76, and pendicular 38: what is the hypothenuse?

$$\text{base} = 76$$
$$\therefore (\text{base})^2 = 76^2 = 5776$$
$$(\text{perp})^2 = 38^2 = 1444$$
$$(\text{base})^2 + (\text{perp.})^2 = 7220 = (\text{hypoth.})^2$$

```
       7220)84·97 = hypoth. Ans.
       64
       ---
  164) 820
       656
       ---
 1689) 16400
       15201
       -----
16987) 119900
       118909
       ------
          991
```

4. The hypothenuse of a right-angled triangle is 109, a
the perpendicular 60: what is the base?

$$\begin{array}{lrl}
(\text{hypoth.})^2 = & 109^2 = & 11881 \\
(\text{perp.})^2 = & 60^2 = & 3600 \\
\hline
(\text{hypoth.})^2 - (\text{perp.})^2 & = & 8281
\end{array}$$

$$\begin{array}{rl}
 & 8\dot{2}8\dot{1}(91 = \text{base, the answer.} \\
 & 81 \\
\hline
181) & 181 \\
 & 181 \\
\hline
\end{array}$$

5. The height of a tower, standing close by the edge of
ditch, is 54 feet, and the breadth of the ditch is 47 feet; r
quired the length of a string that will reach from the top
the tower to the farther side of the ditch.

$$\begin{array}{lrl}
(\text{height})^2 = & 54^2 = & 2916 \\
(\text{breadth})^2 = & 47^2 = & 2209 \\
\hline
(\text{height})^2 + (\text{breadth})^2 & = & 5125
\end{array}$$

$$\begin{array}{rl}
 & 5\dot{1}2\dot{5}(71\cdot58 \text{ feet} = \text{length required,} \\
 & 49 \\
\hline
141) & 225 \\
 & 141 \\
\hline
1425) & 8400 \\
 & 7125 \\
\hline
14308) & 127500 \\
 & 114464 \\
\hline
 & 13036 \\
\hline
\end{array}$$

6. The height of a precipice, standing close by the side
river, is 103 feet, and a line of 320 feet will reach from th
of it to the opposite bank; required the breadth of the ri

$$(\text{height})^2 = 103^2 = 10609$$
$$(\text{hypoth.})^2 = 320^2 = 102400$$

$$(\text{hypoth.})^2 - (\text{height})^2 = 91791$$

$\dot{9}1\dot{7}9\dot{1}(302{\cdot}97$ feet. *Ans.*
9

602) 1791
1204

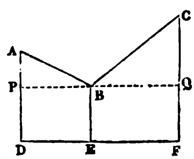
6049) 58700
54441

60587) 425900
424109

1791

7. Suppose a slack rope has its extremities fastened to the tops of two perpendicular poles whose altitudes are 25 and 15 feet, and distance 9 yards. Then if a mountebank walking on the rope, breaks it, and falls 11 feet, find the length of the rope; supposing the mountebank to fall on the ground at a distance of 4 yards from the shorter pole.

Let A B C be the position of the rope; and B the place from which the mountebank fell. Draw B E perpendicular to D F, and through B draw P B Q parallel to D F.



by supposition B E = 11 feet; ∴ D P = 11 feet, and feet. *Also* Q F = 11 feet; therefore C Q = 14 feet.
= 4 *yards* = 12 feet, consequently E F = 15 feet

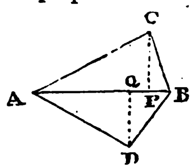
Now $AB = \sqrt{AP^2 + PB^2} = \sqrt{4^2 + 12^2} = \sqrt{160} = 12{\cdot}649$ feet and $CB = \sqrt{CQ^2 + QB^2} = \sqrt{14^2 + 15^2} = \sqrt{421} = 20{\cdot}518$ feet

$\therefore AB + CB = 12{\cdot}649 + 20{\cdot}518 = 33{\cdot}167$ feet, the whole length of the rope.

PROBLEM V.

RULE I.

Ex. 2. Required the area of a trapezium whose diagonal i 70·5 feet, and the two perpendiculars 26·5 and 30·2 feet.



$AB = 70{\cdot}5$; $CP = 26{\cdot}5$

$DQ = 30.2$

56·7 sum of the perp.

```
 70·5
 56.7
 ----
 4935
4230
3525
--------
2)3997.35
--------
```

Ans. 1998·675 *sq. feet.*

Otherwise.

$AB = 70.5$

$\frac{1}{2}CP = 13{\cdot}25$

```
  6625
 92750
```

$\therefore AB \times \frac{1}{2}CP = 934{\cdot}125$ = area of the triangle AC

Also A B = 70·5
$\frac{1}{2}$ D Q = 15·1

755
10570

∴ A B × $\frac{1}{2}$ D Q = 1064·55 = area of the triangle A B D.
Hence 934·125 × 1064·55 = 1998·675 *sq. feet*, as before.

3. What is the area of a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet 9 inches respectively?

Sum of the two perpendiculars = 117·0
108·5

7595
1085
1085

2)12694·5

6347·25 sq. feet.
144

36·

Ans. 6347 *sq. feet*, 36 *sq. inches.*

4. How many square feet are there in a trapezium whose diagonal is 64 feet, and the two perpendiculars 28 and 32 feet?

Sum of the two perpendiculars = 60
64

2)3840

Ans. = 1920 *sq. feet.*

5. *How many acres are there in a field in the shape of a trapezium, whose diagonal is* 4·75 chains, and the two per-·s *let fall* upon it 3·6 and 2·25 chains?

Sum of the two perpendiculars = 5·85
diagonal = 4·75

```
        2925
       4095
      2240
   10)27·7875 sq. chains.
   2) 2·77875
      1·38937
            4
      1·55748
           40
     22·29920
```

Ans. 1 *acre,* 1 *rood,* 22 *perche*

6. How many acres are there in a trapezium, one of who diagonals is 9 chains, and the two perpendiculars let fall upo it, 4·5 and 3·25 chains respectively?

Sum of the two perpendlculars = 7·75
diagonal = 9

```
   10)69·75
   2) 6·975
      3·4875
           4
      1·9500
          40
     38·0000
```

Ans. 3 *acres,* 1 *rood,* 38 *pe*

PROBLEM VI.

Ex. 2. The parallel sides of a trapezoid are 12·41 an
chains; also the perpendicular distance = 5·15 chain
quired its area.

$$
\begin{array}{r}
12.41 \\
8{\cdot}22 \\
\hline
\text{Sum of parallel sides} = 20{\cdot}63 \\
5{\cdot}15 \\
\hline
10315 \\
2063 \\
10315 \\
\hline
2)106{\cdot}2445 \\
\hline
10)53{\cdot}12225 \\
\hline
5{\cdot}312225 \\
4 \\
\hline
1{\cdot}24890\dot{0} \\
40 \\
\hline
9{\cdot}956000 \\
\hline
\end{array}
$$

Ans. 5 *acres*, 1 *rood*, 9 *p*

3. Required the area of a trapezoid whose parallel si
25 feet 6 inches, and 18 feet 9 inches: also the perpe
lar distance is 10 feet 5 inches.

$$
\begin{array}{rrrr}
 & \textit{feet.} & \textit{inches.} & \\
 & 25 & 6 & \\
 & 18 & 9 & \\
\hline
\text{Sum of parallel sides} = & 44 & 3 & \\
 & 10 & 5 & \\
\hline
 & 442 & 6 & \\
 & 10 & 5 & 3 \\
\hline
 & 2)460 & 11 & 3 \\
\hline
 & 330 & 5 & 7 \\
\hline
\end{array}
$$

Ans. 230 *sq. feet*, 5 parts, 7 s

4. Required the area of a trapezoid whose parallel sides are $10\frac{1}{2}$ and $12\frac{1}{4}$ feet, and the perpendicular distance $10\frac{3}{4}$ feet.

$$\begin{array}{rrl}
 & 10\frac{1}{2} = & 10\cdot5 \\
 & 12\frac{1}{4} = & 12\cdot25 \\
\hline
\text{Sum of parallel sides} = & & 22\cdot75 \\
 & 10\frac{3}{4} = & 10\cdot75 = \text{perp. distance.} \\
\hline
 & & 11375 \\
 & & 15925 \\
 & & 2275 \\
\hline
 & & 2)244\cdot5625 \\
\hline
 & \text{Area} = & 122\cdot28125 \textit{ sq. feet.} \quad \textit{Ans.}
\end{array}$$

5. Required the area of a trapezoid whose two parallel sides are 75 and 122 links, and the perpendicular distance 154 links.

$$\begin{array}{rrl}
 & & 75 \\
 & & 122 \\
\hline
\text{Sum of parallel sides} = & & 197 \\
 & & 154 = \text{perp. distance.} \\
\hline
 & & 788 \\
 & & 985 \\
 & & 197 \\
\hline
 & & 2)30338 \\
\hline
 & \text{Area} = & 15169 \textit{ sq. links.} \quad \textit{Ans.}
\end{array}$$

PROBLEM VII.

Ex. 2. *Required the area of a hexagon whose side is 14·6 feet, and perpendicular 12·64* feet.

one side = 14·6
6 = no. of sides.

87·6
12·64

7584
8848
10112

2)1107·264

Area = 553·632 *sq. feet.* *Ans.*

3. Required the area of a heptagon whose side is 1
and perpendicular from the centre = 28.

one side = 19·38
7 = no. of sides.

135·66
28

108528
27132

2)3798·48

1899·24 = *area required.*

4. Required the area of an octagon whose side is 8·
perpendicular 12.

8·9
8 = no. of sides.

71·2
12

2)854·4

Area = 427·2 *as required.*

5. Required the area of a nonagon, one of whose sides = 8 [fee]t, and the perpendicular from the centre = 10·99 feet.

one side = 8 feet.
9 = no. of sides.

72
10·99

2198
7693

2)791·28

Area = 395·64 *sq. feet.* *Ans.*

PROBLEM VIII.

Ex. 2. The side of a regular hexagon is 4 feet 6 inches; [wh]at is its area?

feet.
4·5
4·5

225
180

20·25

2·598076 { area of a hexagon, a side of which is = 1.
20·25

12990380
5196152
5196152

52·61103900 *Ans.*

3. Find the area of a regular octagon, one of wl
15 feet.

15
15
—
75
15
—
225
—

4·828427 = { area of an octa
225 { of which is
—
24142135
9656854
9656854
—
Area 1086·396075 *sq. feet.* *Ans.*

4. Required the area of a regular decagon, on
sides is = 20·5 yards.

20·5
20·5
—
1025

—
420·25
—

7·694208 = { area of a decag
420·24 { of which is
—
38471040
15388416
15388416
30776832
—
Area 3233·49091200 *sq. yards.* Ans.

Find the area of a regular nonagon, one of whose sides
et.

$$
\begin{array}{r}
8 \\
8 \\
\hline
64 \\
\hline
6{\cdot}181824 \\
64 \\
\hline
24727296 \\
37090944 \\
\hline
395{\cdot}636736
\end{array}
\quad = \begin{cases} \text{area of a nonagon, a side} \\ \text{of which is} = 1. \end{cases}
$$

395·636736 *sq. feet.* *Ans.*

Find the area of a regular dodecagon, one of whose sides
et.

$$
\begin{array}{r}
6 \\
6 \\
\hline
36 \\
\hline
11{\cdot}196152 \\
36 \\
\hline
67176912 \\
33588456 \\
\hline
403{\cdot}061472
\end{array}
\quad = \begin{cases} \text{area of a dodecagon, a side} \\ \text{of which is} = 1. \end{cases}
$$

403·061472 *sq. feet.* *Ans.*

PROBLEM IX.

2. Find the area of the annexed figure, in which the dimensions have been taken as follows:

B

$AE = 6{\cdot}4$ yards.
$Fx = 1{\cdot}5$
$By = 2{\cdot}3$
$BE = 5{\cdot}1$
$Dz = 2{\cdot}8$
$CD = 6{\cdot}3$
$Bm = 2{\cdot}2$

$\frac{1}{2}\,AE \cdot Fx = 4{\cdot}8$
$\frac{1}{2}\,AE \cdot By = 7{\cdot}36$
$\frac{1}{2}\,BE \cdot Dz = 7{\cdot}14$
$\frac{1}{2}\,CD \cdot Bm = 6{\cdot}93$

$26{\cdot}23$ sq. yds. = area of the figure A B C D E F.

3. Since similar rectilineal figures are to one another as the squares of their homologous sides, therefore the area of figure in Example 1. is to area of figure in this Example : : $1 : (\frac{1}{2})^2$: : $4 : 1$; hence the area of figure in this Example $= \frac{1}{4}$ of area in Example 1 $= \frac{1}{4} \times 521{\cdot}525 = 130{\cdot}381$ = answer required.

4. In this case $Ba = 12{\cdot}5$ feet.
$Ea = 16{\cdot}$

$\therefore BE = 28{\cdot}5$ = base of triangle A B E.
$Aa = 10{\cdot}$

2)285

$142{\cdot}5$ = area of $\triangle$ A B C.
$\frac{1}{2} \cdot BC \times Cc = 28{\cdot}$ = area of $\triangle$ C B c.
$\frac{1}{2} \cdot Dd \times Ed = 21{\cdot}$ = area of $\triangle$ D d E.
$(Cc + Dd) \times \frac{dc}{2} = 116{\cdot}25$ = area of C c d D.

$307{\cdot}75$ = area of figure A B C D E.

5. Find the area of an irregular hexagonal figure A B C D E F, supposing the following dimensions to be given: *viz.* $Am =$ 4 yards, $mF = 6$ yards, $En = 6\frac{1}{2}$ yards, $An = 9$ yards, $AD =$ *14 yards*, $Ax = 10$ yards, $Ab = 6{\cdot}5$ yards, $Bb = 7$ yards, and $xc = 6\frac{3}{4}$ yards.

$\frac{1}{2}$ Am · mF $= 12·$ $=$ area of triangle AmF.

$+$ En) · $\frac{mn}{2} = 31·25$ $=$ area of trapezium FmnE.

$\frac{1}{2}$ Dn · nE $= 16·25$ $=$ area of triangle DnE.

$\frac{1}{2}$ Ab · Bb $= 22·75$ $=$ area of triangle AbB.

$+$ Cx) · $\frac{bx}{2} = 24·0625$ $=$ area of trapezium BbxC.

$\frac{1}{2}$ Dx · xC $= 13·50$ $=$ area of triangle DxC.

119·8125 $=$ area of figure A B C D E F.

PROBLEM X.

RULE I.

Ex. 3. The length of an irregular figure is 125 yards, and breadths at 15 equi-distant places are 5·2, 4·6, 7·2, 8·3, , 8·1, 7·3, 7·9, 6·6, 7·2, 7·3, 8·4, 7·4, 6·5, 5·8; what is its a ?

n of the measured breadths $=$ 107·2

15) 7·1466

125

357330

142932

71466

Area $=$ 893·3250 *sq. yards.*

RULE I.

Ex. 2. The length of an irregular figure is 39 yards, and its adths *in five equi-distant* places are 4·8, 5·2, 4·1, 7·3, and ; *what is its area?*

$$
\begin{array}{l}
4 \cdot 8 + 7 \cdot 2 = 12 = \text{sum of extre}\\
\therefore 6 = \tfrac{1}{2} \text{ sum of extreme b}\\
5 \cdot 2\\
4 \cdot 1\\
7 \cdot 3\\
\hline
4)22.6\\
\hline
5 \cdot 65 = \text{mean breadth.}\\
39\\
\hline
5085\\
1695\\
\hline
\text{Area} = 220 \cdot 35 \textit{ sq. yards.}
\end{array}
$$

3. The length of an irregular figure is 50 yai breadths at seven equi-distant places are 5·5, 6·2, 7·, and 8·6; what is its area?

$$
\begin{array}{l}
5 \cdot 5 + 8 \cdot 6 = 14 \cdot 1 \text{ sum of extrer}\\
\therefore 7 \cdot 05 = \tfrac{1}{2} \text{ sum of extreme breadth:}\\
6 \cdot 2\\
8 \cdot 7\\
6 \cdot\\
7 \cdot 5\\
7 \cdot\\
\hline
6)41 \cdot 05\\
\hline
6 \cdot 8416\\
50\\
\hline
\text{Area} = 342 \cdot 0800 \textit{ sq. yards.}
\end{array}
$$

4. The solution is given in the Mensuration.

5. Find the area of an irregular figure, like that supposing A B = 52 and A D = 23 yards, and that *six perpendicular* breadths of the curvilinear space *from* D C, viz. 11·5, 10·2, 8·3, 9·8, 10·1, and 12· *ectively*.

$$
\begin{array}{rl}
11{\cdot}5 & \\
12{\cdot}6 & \\
\hline
2)24{\cdot}1 & = \text{sum of extreme breadths.} \\
\hline
12{\cdot}05 & = \tfrac{1}{2}\text{ sum of extreme breadths.} \\
10{\cdot}2 & \\
8{\cdot}3 & \\
9{\cdot}8 & \\
10{\cdot}1 & \\
\hline
5)50{\cdot}45 & \\
\hline
3)10{\cdot}09 & = \text{mean breadth in feet.} \\
\hline
3{\cdot}3633 & = \text{mean breadth in yards.} \\
52 & \\
\hline
67266 & \\
168165 & \\
\hline
174{\cdot}8916 & = \text{area of curvilinear space.} \\
1196{\cdot} & = \text{area of the rectangle D B.} \\
\hline
\text{Area} = 1370{\cdot}8916 & \textit{sq. yards.} \\
\hline
\end{array}
$$

PROBLEM XI.

The diameter of a circle being given, to find the circumference; the circumference being given, to find the diameter.

RULE I.

Ex. 3. If the circumference of a circle be 48 feet, what is length of its diameter?

$$
\begin{array}{r}
\text{As } 22 : 7 :: 48 \\
7 \\
\hline
22)336 \\
\hline
15\tfrac{3}{11}\ \textit{feet} = \text{the diameter, nearly.}
\end{array}
$$

4. If the diameter of a circle be 15 feet, what is the of its circumference?

$$
\begin{array}{r}
\text{As } 7 : 22 :: 15 \\
22 \\
\hline
30 \\
30 \\
\hline
7)330 \\
\hline
47\tfrac{1}{7}
\end{array}
$$

$47\frac{1}{7}$ *feet* = the circumference, I

RULE II.

Ex. 3. The diameter of a circle is 7 yards, what is th of its circumference?

$$
\begin{array}{r}
3{\cdot}1416 \\
7 \\
\hline
\end{array}
$$

Circumference $= 21{\cdot}9912$ *yards*.

4. The circumference of a circle is 50 feet, what is th of the diameter?

$$
\begin{array}{l}
3{\cdot}1416)50{\cdot}000000(15{\cdot}915, \text{ \&c.} = \text{d} \\
\phantom{3{\cdot}1416)}31416 \\
\phantom{3{\cdot}1416)}\overline{185840} \\
\phantom{3{\cdot}1416)}157080 \\
\phantom{3{\cdot}1416)}\overline{287600} \\
\phantom{3{\cdot}1416)0}282744 \\
\phantom{3{\cdot}1416)00}\overline{48560} \\
\phantom{3{\cdot}1416)000}31416 \\
\phantom{3{\cdot}1416)000}\overline{171440} \\
\phantom{3{\cdot}1416)000}157080 \\
\phantom{3{\cdot}1416)000}\overline{14360}
\end{array}
$$

f the diameter of a circle be 34 feet, what is the length
circumference?

3·1416
34

125664
94248

rcumference = 106·8144 *feet.*

: the circumference of a circle be 177 yards, what is the
of the diameter?

3·1416)177.00000(56·34 *yards* = the diameter.
157080

199200
188496

107040
94248

127920
125664

2256

the circumference of the earth be supposed equal to
niles, what is the earth's diameter?

3·1416)25000·0000(7957·72, &c.
219912

300880
282744

181360
157080

242800
219912

228880
219912

89680
62832

26848

If 25000 be divided by 3·1415, the quotient is 795
now the circumference of a circle to diameter 1 is eq
3·14159265, &c. which is less than 3·1416, but greater
3·1415; hence the nearest integer which expresses the e
diameter is 7958 miles; the circumference being suppos
25000 miles.

PROBLEM XII.

To find the length of any circular arc.

RULE I.

Ex. 3. The chord of an arc is 36·75, and the chord o
the arc 23·2; what is the length of the arc?

$$\begin{array}{rl}
23\cdot2 & = \text{chord } \tfrac{1}{2} \text{ arc.} \\
8 & \\
\hline
185\cdot6 & \\
36\cdot75 & = \text{chord of whole a} \\
\hline
3)148\cdot85 & \\
\hline
\text{Length required} = 49\cdot616, \&c. & \\
\hline
\end{array}$$

4. The chord of the whole arc is 16, and the chord o
the arc 9·5; what is the length of the arc?

$$\begin{array}{rl}
9\cdot5 & = \text{chord of } \tfrac{1}{2} \text{ arc.} \\
8 & \\
\hline
76\cdot0 & \\
16 & \\
\hline
3)60\cdot & \\
\hline
\text{Length required} = 20\cdot & \\
\hline
\end{array}$$

5. The chord of the whole arc is 40, and the chord
the arc 24·5; what is the length of the arc?

24·5 = chord of ½ arc.
8

196·0
40·

3)156

Ans. = 52 = length of arc as required.

'he chord of the whole arc is 7 feet, and the chord of e arc 4 feet; what is the length of the arc?

4 = chord of ½ arc.
8

32
7 = chord of whole arc.

3)25

Length required = 8·333 *feet.*

RULE II.

2. Required the length of an arc of $55^0\ 30'$; the dia- of the circle being 25 feet.

$55^0\ 30' = 55·5$
12·5 = radius.

2775
1110
555

693·75
·0174533

208125
208125
346875
277500
485625
69375

gth of arc = 12·108226875 *feet.*

3. Required the length of an arc of $57° 17' 44\frac{1}{2}''$; meter of the circle being 50 feet.

6,0)44·5

6,0)17·7416

57·29569 degrees.

25 = radius.

28647845

11459138

1432·39225

·0174533

429717675

429717675

716196125

572956900

1002674575

143239225

24·999971656925 = 25 *feet*, very near radius of the cir

4. Required the length of an arc of $45° 30' 30''$ dius of the circle being 20 feet.

$45° 30' 30'' = 45·5083$

20 = radius.

91·01660

·0174533

27304980

2730498

4550830

3640664

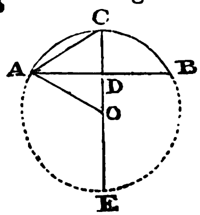
6371162

910166

Length = 158854·0024780 feet.

RULE III.

x. 2. The chord of the whole arc A C B is $48\frac{1}{2}$ feet, and eight C D is $18\frac{1}{4}$; find the length of the arc.



$$A\,D\,B = 48{\cdot}5$$
$$\therefore A\,D = 24{\cdot}25$$

ence $A\,C = (A\,D^2 + D\,C^2)^{\frac{1}{2}} = \sqrt{(24{\cdot}25)^2 + (18{\cdot}25)^2}$
$= (588{\cdot}0625 + 333{\cdot}0625)^{\frac{1}{2}}$
$= (921{\cdot}125)^{\frac{1}{2}} = 30{\cdot}35$ feet.

Hence $30{\cdot}35$ = chord of $\frac{1}{2}$ the arc A C B.*

$$\begin{array}{r} 8 \\ \hline 242{\cdot}80 \\ 48{\cdot}5 \\ \hline 3)194{\cdot}3 \\ \hline \end{array}$$

Length of arc = $64{\cdot}767$ *feet*, nearly.

The chord of the whole arc is 7 feet, and the versed sine, titude C D = 2; find the length of the arc.

Here A D = $3{\cdot}5$ feet.

D C = 2 feet.

$(A\,D^2 + D\,C^2)^{\frac{1}{2}} = (12{\cdot}25 + 4)^{\frac{1}{2}} = (16{\cdot}25)^{\frac{1}{2}} = 4{\cdot}031 = A\,C.$

Hence $4{\cdot}031$ = chord of $\frac{1}{2}$ arc A C B.

$$\begin{array}{r} 8 \\ \hline 32{\cdot}248 \\ 7 \\ \hline 3)25{\cdot}248 \end{array}$$

Length = $8{\cdot}416$ *feet*.

* See Rule I. in this Problem.

4. The chord of the whole arc is 40, and the altitude C I 15; what is the length of the arc?

$$\text{A D} = 20$$
$$\text{D C} = 15$$
$$\because (\text{A D}^2 + \text{D C}^2)^{\frac{1}{2}} = (625)^{\frac{1}{2}} = 25 = \text{A C} = \text{chord of half the a}$$

$$\begin{array}{r} 25 \\ 8 \\ \hline 200 \\ 40 \\ \hline 3)160 \\ \hline \end{array}$$

Length = 53·33, &c.

5. The chord of the whole arc is 20·386, and the hei C D = 4; what is the length of the arc?

Here $\text{A D} = 10{\cdot}193$

$$\therefore \text{A C} = (\text{A D}^2 + \text{D C}^2)^2 = (103{\cdot}897249 + 16)^{\frac{1}{2}} = (119{\cdot}897249)^{\frac{1}{2}} = 10{\cdot}949.$$

$10{\cdot}949 = \text{chord of } \frac{1}{2} \text{ the arc.}$

$$\begin{array}{r} 8 \\ \hline 87{\cdot}592 \\ 20{\cdot}386 \\ \hline 3)67{\cdot}206 \\ \hline \end{array}$$

Length required = 22·402 as required.

PROBLEM XIII.

To find the area of a circle whose diameter is given.

RULE I.

hat is the area of a circle whose diameter is 7

$$\begin{array}{r}
\cdot 7854 \\
49 \\
\hline
70686 \\
31416 \\
\hline
38\cdot 4846 \\
\hline
\end{array}
\begin{array}{l}
\\
= 7^2 = (\text{diameter})^2 \\
\\
\\
\\
= \text{area required.}
\end{array}$$

Required the area of a circle whose radius is 2·25.

$$\begin{array}{r}
\cdot 7854 \\
20\cdot 25 \\
\hline
39270 \\
15708 \\
15708 \\
\hline
15\cdot 904350 \\
\hline
\end{array}
\begin{array}{l}
\\
= (4\cdot 5)^2 = (\text{diameter})^2 \\
\\
\\
\\
\\
= \text{area required.}
\end{array}$$

How many square yards are there in a circle whose dia-
: is 3·5 feet?

$$\begin{array}{r}
\cdot 7854 \\
12\cdot 25 \\
\hline
39270 \\
15708 \\
15708 \\
7854 \\
\hline
9)9\cdot 621150 \\
\hline
\end{array}
\begin{array}{l}
\\
= (3\cdot 5)^2 = (\text{diameter})^2 \\
\\
\\
\\
\\
\\
\text{sq. feet.}
\end{array}$$

Area = 1·069016 sq. yards.

How many square yards are contained in a circle whc
er is 30·5 feet?

$$
\begin{array}{r}
\cdot 7854 \\
930\cdot 25 = (30\cdot 5)^2 = (\text{diameter})^2 \\
\hline
39270 \\
15708 \\
23562 \\
70686 \\
\hline
9)730\cdot 618350 \text{ sq. feet.} \\
\hline
\text{Area} = \underline{81\cdot 179816} \text{ sq. yards.}
\end{array}
$$

6. Determine the radius of a circular field containing acre of meadow.

Now one acre contains 4840 sq. yards.

Let $\triangle$ = diameter of the required circle, in yards.

Then $\cdot 7854 \times \triangle^2 = 4840$.

and $\triangle^2 = .\frac{4840}{7854} = 6162\cdot 4649$.

$\therefore \triangle = \sqrt{6162\cdot 4649} = 78\cdot 5 = 78\frac{1}{2}$ yards, nearly;

Consequently radius required $= 39\frac{1}{4}$ *yards,* nea

7. Determine the radius of a circle whose area is equal one rood.

In this case $\cdot 7854 \times \triangle^2 = 1210$, being $\frac{1}{4}$ of 4840

$\therefore \triangle^2 = .\frac{1210}{7854} = 1540\cdot 616$

$\therefore \triangle = \sqrt{1540\cdot 616} = 39\cdot 25 = 39\frac{1}{4}$ yard

Hence the radius required $= 19\frac{5}{8}$ *yards*

8. Find the radius of a circle whose area is equal to that a square whose side is 22 yards.

$$
\begin{array}{r}
22 \\
22 \\
\hline
44 \\
44 \\
\hline
484 = \text{area of the square.}
\end{array}
$$

Also, if $\triangle$ = the required diameter of the circle,
$\cdot7854 \times \triangle^2$ = area of the circle = 484 by supposition

$$\therefore \triangle^2 = \frac{484}{\cdot 7854} = 616\cdot2465, \text{ nearly};$$

$$\therefore \triangle = 24\cdot824 \text{ yards} = \text{the diameter};$$

$$\therefore \text{radius} = 12\cdot412 \textit{ yards}, \text{ nearly.}$$

RULE II.

To find the area of a circle whose circumference is given.

Ex. 2. Find the area of a circle whose circumference is feet.

```
        22
        22
        --
        44
       44
       ---
       484
    ·07958
    ------
     31832
    63664
   31832
   ------
```

Area required = 38·51672 *sq. feet.*

3. Required the area of a circle whose circumference is 9

```
       9·5
       9·5
       ---
       475
      855
      ---
     90·25
    ·07958
    ------
     72200
    45125
   81225
  63175
  -------
```

Area required = 7·1820950

4. How many square feet are there in a circle whose cumference is 20·1 yards?

```
          20·1
          20·1
          ----
          20·1
         402
         ------
         404·01
         ·07958
         ------
         323208
        202005
       363609
      282807
      ----------
      32·1511158 sq. yards.
               9
      ----------
     289·3600422 sq. feet.
```

5. Find the area of a circle whose circumference is one fu long.

```
   8)1760 yards = 1 mile.
     ----
      220 yards = one furlong.
      220
      ----
     4400
    44
    -----
    48400 = (circumf.)²
    ·07958
    ------
     31832
    63664
   31832
   ------
d = 3851·672 sq. yards.
```

6. Determine the number of acres in a circle whose circumference is one mile.

$$
\begin{array}{r l}
1760 & \text{yards} = \text{one mile.} \\
1760 & \\
\hline
105600 & \\
1232 & \\
176 & \\
\hline
3097600 & = (\text{circumference})^2 \\
\cdot 07958 & \\
\hline
24780800 & \\
154880 & \\
278784 & \\
216832 & \\
\hline
4840)246507 \cdot 00800 & \text{sq. yards.} \\
\hline
50 \cdot 9312 & \text{acres.} \\
4 & \\
\hline
3 \cdot 7248 & \text{roods.} \\
40 & \\
\hline
28 \cdot 9920 & \\
\hline
\end{array}
$$

Ans. 50 *acres,* 3 *roods,* 29 *poles,* nearly.

PROBLEM XIV.

To find the area of a circular ring, or the space included between the circumferences of two concentric circles.

RULE I.

Ex. 2. The diameters of two concentric circles are 15 and 10; *what is the area of* the ring contained between their circumferences?

$$\begin{array}{r} 10 \\ 10 \\ \hline 100 \\ \cdot 7854 \\ \hline 78\cdot 54 \end{array} = \text{area of smaller circle.}$$

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ \cdot 7854 \\ \hline 39270 \\ 15708 \\ 15708 \\ \hline \end{array}$$

$$\begin{array}{rr} \text{Area of larger circle} = & 176\cdot 7150 \\ & 78\cdot 54 \\ \hline \text{Area required} = & 98\cdot 175 \\ \hline \end{array}$$

3. Required the area of the ring, the diameters of w: bounding circles are 9 and 5 inches.

$$\begin{array}{r} 5 \\ 5 \\ \hline 25 \\ \cdot 7854 \\ \hline 39270 \\ 15708 \\ \hline 19\cdot 6350 \end{array}$$

$$\begin{array}{r} 9 \\ 9 \\ \hline 81 \\ \cdot 7854 \\ \hline 7854 \\ 62832 \\ \hline \end{array}$$

$$\begin{array}{rr} \text{a of larger circle} = & 63\cdot 6174 \\ & 19\cdot 6350 \\ \hline \text{Area required} = & 43\cdot 9824 \\ \hline \end{array}$$

. The two diameters are 21·25 feet and 9·75 feet; requi area of the ring.

$$\begin{array}{r} 9\cdot 75 \\ 9\cdot 75 \\ \hline 4875 \\ 6825 \\ 8775 \\ \hline 95\cdot 0625 \\ \cdot 7854 \\ \hline 3802500 \\ 4753125 \\ 7605000 \\ 6654375 \\ \hline 74\cdot 66208750 \\ \hline \end{array}$$

```
                    21·25
                    21·25
                   ------
                   10625
                   4250
                  2125
                 4250
                 --------
                 451·5625
                    ·7854
                 --------
                 18062500
               22578125
             36125000
           31609375
           ------------
Area of larger circle = 354·65718750
                         74·66208750
                        ------------
    Area required = 279·9951
```

5. The diameters of two concentric circles are 15 and feet; required the area of the ring.

```
        15
        15
        --
        75
       15
       ---
       225
     ·7854
     -----
     39270
    15708
   15708
  --------
  176·7150
```

```
                       16
                       16
                       --
                       96
                      16
                      ---
                      256
                     ·7854
                     -----
                    47124
                   39270
                  15708
                  --------
Area of larger circle = 201·0624
                  176·715
                  --------
                   24·3474 = area of the ring.
                  --------
```

RULE II.

Ex. 2. The diameters of two concentric circles are 5 and 5·5 feet; find the area of the ring.

```
Sum of diameters = 10·5
Difference of do. =    5
                     ----
                     52·5
                    ·7854
                    -----
                    39270
                  15708
                 39270
                 -------
       Area = 4·123350 sq. feet.
                 -------
```

3. The diameters of two concentric rings are 6·5 and 4·9 feet respectively; find the area of the ring.

Sum of diameters = 11·4
Difference of do. = 1·6

684
114

18·24
·7854

7296
9120
14592
12768

Area required = 14·325696 *sq. feet.*

PROBLEM XV.

To find the area of a sector of a circle.

RULE I.

Ex. 2. The chord of an arc is 28 feet, the chord of half arc is 16 feet; what is the area of the sector?

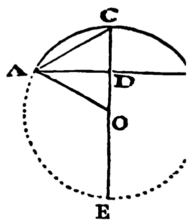
Here AB=28 ∴ AD=14
and AC=16
8

128=8, chord $\frac{1}{2}$ arc.
28

3)100

2) 33·33

16·67=$\frac{1}{2}$ length of arc A C B.



Now $CD = (CA^2 - AD^2)^{\frac{1}{2}} = (16^2 - 14^2)^{\frac{1}{2}} = (256 - 96)^{\frac{1}{2}} =$ ∿ $= 7·746$. But $EC \cdot CD = CA^2$, ∴ $EC \times 7·746 = 256$ ∴ $EC = \frac{256}{7·746} = 33·05$, nearly; ∴ radius $OC = 16·52$. N *the area of the sector* is equal to the radius multiplied by $\frac{1}{2}$ *le* · *arc* $= 16·52 \times 16·67 = 275·3884$ sq. feet.

3. Required the area of a sector of a circle, whose radius is 25, and the length of the arc 21·5.

$$\begin{array}{rl} 2)21\cdot5 & \\ \hline 10\cdot75 & = \tfrac{1}{2} \text{ length of the arc.} \\ 25 & = \text{radius} \\ \hline 5375 & \\ 2150 & \\ \hline 268\cdot75 & = \text{area of sector.} \end{array}$$

4. What is the area of a sector whose radius is 10 feet, the chord of the whole arc 18·372, and the chord of half the arc 11 feet?

$$\begin{array}{rl} \text{chord of } \tfrac{1}{2} \text{ arc} = & 11 \\ & 8 \\ \hline & 88\cdot \\ \text{chord of whole arc} = & 18\cdot372 \\ \hline & 3)69\cdot628 \\ \hline \text{length of arc} = & 23\cdot209 \\ & 2)\overline{} \\ & 11\cdot6045 \\ & 10 \\ \hline \text{Area of sector} = & 116\cdot045 \end{array}$$

5. Find the area of the sector whose radius is 9, and the chord of whose arc is 6.

$$\text{A D} = 3$$
$$\text{A O} = 9$$
$$\therefore \text{O D} = (\text{A O}^2 - \text{A D}^2)^{\frac{1}{2}} = (81 - 9)^{\frac{1}{2}} = (72)^{\frac{1}{2}} = 8\cdot485.$$

Hence C D $= 9 - 8\cdot485 = \cdot515$.

Also A C $= (\text{A D}^2 + \text{D C}^2)^{\frac{1}{2}} = (9 + \cdot265225)^{\frac{1}{2}} = (9\cdot265225)^{\frac{1}{2}} =$ *3·044, nearly.*

$$
\begin{array}{r l}
3{\cdot}044 & = \text{chord of } \tfrac{1}{2} \text{ the arc.} \\
8 & \\
\hline
24{\cdot}352 & \\
6 & \\
\hline
3)18{\cdot}352 & \\
\hline
6{\cdot}117 & = \text{length of the arc ACB.} \\
4{\cdot}5 & \\
\hline
30585 & \\
24468 & \\
\hline
\end{array}
$$

Area of sector $= 27{\cdot}5265$, nearly.

6. The chord of the whole arc is 16, and its height 6; wh is the area of the sector?

$$AD = 8$$
$$DC = 6$$
$$\therefore AC = (AD^2 + DC^2)^{\frac{1}{2}} = (64+36)^{\frac{1}{2}} = 1$$

Hence 10 = chord of ½ the arc.

$$
\begin{array}{r l}
8 & \\
\hline
80 & = 8 \text{ times chord of } \tfrac{1}{2} \text{ arc.} \\
16 & = \text{chord of whole arc.} \\
\hline
3)64 & \\
\hline
21{\cdot}333 & = \text{length of arc, nearly.} \\
\hline
\end{array}
$$

Now $EC \times CD = CA^2$

i. e. $EC \times 6 = 100$

$\therefore EC = \frac{100}{6} = 16{\cdot}66 =$ the diameter.

and radius $= 8{\cdot}333$

Now the ½ rad. × length of the arc = the area.

$\therefore 4{\cdot}166 \times 21{\cdot}333 = 88{\cdot}873278 =$ the area, nearl

7. *Find the area of a sector whose height is 4, and the ra dius of the circle 8 feet.*

Here $CD = 4$

$\therefore DO = 4$

Also $AD = (AO^2 - DO^2)^{\frac{1}{2}} = (64 - 16)^{\frac{1}{2}} = (48)^{\frac{1}{2}} = 6{\cdot}928.$

$\therefore$ chord of whole arc $= 13{\cdot}856$.

Also $QC \cdot CD = CA^2$, or $16 \times 4 = CA^2 \therefore CA = 8 =$ chord of $\frac{1}{2}$ the arc.

$$\begin{array}{r l}
8 & \\
8 & \\
\hline
64 & = \text{chord } \tfrac{1}{2} \text{ arc.} \\
13{\cdot}856 & = \text{chord of whole arc.} \\
\hline
3)50{\cdot}144 & \\
\hline
16{\cdot}714 & = \text{length of the arc, nearly.} \\
4 & = \tfrac{1}{2} \text{ radius.} \\
\hline
66{\cdot}856 & = \text{area of sector.} \\
\hline
\end{array}$$

RULE II.

Ex. 2. Find the area of a sector, the arc of which contains 30°, and the length of the diameter 3 feet.

$$\begin{array}{r l}
{\cdot}7854 & = \text{area of a circle to diameter 1.} \\
9 & \\
\hline
7{\cdot}0686 & = \text{area of whole circle to diameter 3 feet.}
\end{array}$$

Hence 360° : 30° : : 7·0686 : area required.
or 12 : 1 : : 7·0686 : area required.

$$\therefore \text{area required} = \frac{7{\cdot}0686}{12} = {\cdot}58905, \textit{ sq. feet.}$$

3. Determine the area of a sector whose arc contains 147° 29′, and the length of its radius 25 feet.

$$\begin{array}{r l}
{\cdot}7854 & = \text{area of a circle to diameter 1.} \\
2500 & = (\text{diameter})^2 \\
\hline
3927000 & \\
15708 & \\
\hline
1963{\cdot}5000 & = \text{area of whole circle.}
\end{array}$$

Hence $360^0 :: 147^0\ 29' :: 1963{\cdot}5$: area required.
or $21600' : 8849' :: 1963{\cdot}5$: area required.

```
           8849
         ------
         176715
          78540
        157080
       157080
      ---------
21600)17375011·5(804·3986 = area r
       172800
      --------
         95011
         86400
        -------
          86115
          64800
         -------
          213150
          194400
         --------
           187500
           172800
          --------
            147000
            129600
           --------
             17400
```

4. Find the area of a quadrant whose radius = 21 f

```
        ·7854 = area of circle to dia
        1764 = 42² = (diameter)
       ------
        31416
       47124
      54978
      7854
     ----------
     1385·4456 = area of whole cir
```

Hence $360^0 : 90^0 :: 1385{\cdot}4456$: area required.
$4 : 1 :: 1385{\cdot}4456$: area required.

$\therefore$ area required $= \frac{1}{4} \times 1385{\cdot}4456 = 346{\cdot}3614$

'equired the area of the sector whose arc contains 17°
d the diameter 19 feet.

·7854 = area of a circle to diameter 1.
361 = 19^2 = $(\text{diameter})^2$

7854
47124
23562

283·5294 = area of whole circle.

e 360° : 17°25 :: 283·5294 : area required.
17·25

14176470
5670588
19847058
2835294

360)4890·882150(13·585, &c. *sq. feet.*
360

1290
1080

2108
1800

3088
2880

2082
1800

282

PROBLEM XVI.

To find the area of a segment of a circle.

RULE I.

1. What is the area of the segment of a circle whose dia-
s 18 *feet long; the arc* of the segment containing 90°?

$$
\begin{array}{rl}
\cdot 7854 & = \text{area of a} \bigcirc \text{ to dian} \\
324 & = 18^2 = (\text{diameter})^2 \\
\hline
31416 & \\
15708 & \\
23562 & \\
\hline
4)254\cdot 4696 & = \text{area of whole circl} \\
\hline
63\cdot 6174 & = \text{area of the sector.}
\end{array}
$$

Also the area of the rectilineal triangle $= \frac{81}{2} = 40\cdot 5$

$\therefore 63\cdot 6174 - 40\cdot 5 = 23\cdot 1174$ s

4. Required the area of a segment whose height is chord 20.

$\text{AD} = 10$

$\therefore \text{AC} = (100+4)^{\frac{1}{2}} = (104)^{\frac{1}{2}} = 10\cdot 198$ chord of $\frac{1}{2}$ the

$$
\begin{array}{rl}
10\cdot 198 & \\
8 & \\
\hline
81\cdot 584 & = 8 \text{ chord } \tfrac{1}{2} \text{ arc.} \\
20\cdot & = \text{chord of whole ar} \\
\hline
3)61\cdot 584 & \\
\hline
20\cdot 528 & \text{nearly} = \text{length of th}
\end{array}
$$

Again $\text{QC} \times 2 = 104 \therefore \text{QC} = 52 =$ the diameter.

$$
\begin{array}{lr}
\text{Length of arc} = & 20\cdot 528 \\
\tfrac{1}{2} \text{ radius} \quad = & 13 \\
\hline
& 61584 \\
& 20528 \\
\hline
\text{Area of sector} = & 266\cdot 864 \\
\text{Area of } \triangle \text{AOB} = & 240\cdot \\
\hline
\text{Area of segment} = & 26\cdot 864
\end{array}
$$

5. *Required* the area of a segment whose radius is

-d of half the arc 10·2

$QC \times CD = CA^2$

$$\therefore CD = \frac{CA^2}{QC} = \frac{104 \cdot 04}{48} = 2 \cdot 1675$$

Hence $OD = OC - CD = 21 \cdot 8325$

Also $AD = (AC^2 - CD^2)^{\frac{1}{2}} = 9 \cdot 967.$

$$\begin{array}{ll}
10 \cdot 2 & \\
8 & \\
\hline
81 \cdot 6 & = \text{8 times chord of } \tfrac{1}{2} \text{ arc.} \\
19 \cdot 934 & = \text{chord of whole arc.} \\
\hline
3)61 \cdot 666 & \\
\hline
20 \cdot 555 & = \text{length of arc.} \\
12 & = \tfrac{1}{2} \text{ radius.} \\
\hline
246 \cdot 660 & = \text{area of sector } AOB. \\
217 \cdot 6045 & = \text{area of } \triangle\, ADBO. \\
\hline
29 \cdot 0555 & = \text{area of segment.} \\
\hline
\end{array}$$

. Required the area of a segment of a circle whose chord 2, and the radius of the circle 10.

$AD = 6$

$AO = 10$

$\therefore OD = (AO^2 - AD^2)^{\frac{1}{2}} = (100 - 36)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8.$

and $CD = 2.$

ikewise $QC \cdot CD = CA^2$

i. e. $20 \times 2 = CA^2$

$$\begin{array}{ll}
CA = \sqrt{40} = 6 \cdot 324 & = \text{chord of } \tfrac{1}{2} \text{ the arc.} \\
8 & \\
\hline
50 \cdot 592 & = \text{8 times chord of } \tfrac{1}{2} \text{ arc.} \\
12 \cdot & = \text{chord of whole arc.} \\
\hline
3)38 \cdot 592 & \\
\hline
12 \cdot 864 & = \text{length of the arc.} \\
5 & = \tfrac{1}{2} \text{ radius.} \\
\hline
64 \cdot 320 & = \text{area of sector.} \\
48 \cdot & = \text{area of } \triangle\, ACB. \\
\hline
16 \cdot 32 & = \text{area of segment } ACBDA. \\
\hline
\end{array}$$

7. Determine the area of the segment of a circle whc dius = 32 feet; the arc of the segment being supposed t tain 120^0.

$$
\begin{array}{rl}
3{\cdot}1416 & = \text{area of a circle to radius} \\
1024 & = 32^2 = (\text{radius})^2 \\
\hline
125664 & \\
62832 & \\
31416 & \\
\hline
3216{\cdot}9984 & = \text{area of whole circle.}
\end{array}
$$

Now $360^0 : 120^0 :: 3216{\cdot}9984$: area of sector.

or $3 : 1 :: 3216{\cdot}9984$: area of sector.

$\therefore$ area of sector $= 1072{\cdot}3328$.

Now the chord of $120^0 = 2$ sin. $60^0 = 2$ rad. $\frac{\sqrt{3}}{2} 32\sqrt{3} = 5$

$\therefore$ OD $= (OA^2 - AD^2)^{\frac{1}{2}} = \{32^2 - (27{\cdot}712)^2\}^{\frac{1}{2}} = \{1024$ $767{\cdot}954944\}^{\frac{1}{2}} = (256{\cdot}045056)^{\frac{1}{2}} = 16$, nearly. Henc OD $= 27{\cdot}712 \times 16 = 443{\cdot}392 =$ area of rectilineal t AOB.

$$
\begin{array}{rl}
1072{\cdot}3328 & = \text{area of sector AO} \\
443{\cdot}392 & = \text{area of } \triangle \text{ AOB.} \\
\hline
628{\cdot}9408 & = \text{area of seg.}
\end{array}
$$

RULE II.

Ex. 3. Find the area of a circular segment, whose sine is 2 inches, and the diameter of the circle 52.

Here $\frac{2}{52} = {\cdot}038\frac{6}{13}$ the tabular ver. sin.

$$
\begin{array}{rl}
\text{Tabular area to } {\cdot}038 & = {\cdot}009763 \\
\text{———— } {\cdot}039 & = {\cdot}010148 \\
\hline
\text{Difference} & = {\cdot}000385 \\
& \phantom{= {\cdot}00038}6 \\
\hline
13) & {\cdot}002310 \\
\hline
\text{Part corresp. to } \frac{6}{13} & = {\cdot}000178 \\
\text{Tab. area to } {\cdot}038 & = {\cdot}009763 \\
\hline
\text{Tab. area to } {\cdot}038\frac{6}{13} & = {\cdot}009941 \\
& \phantom{= {\cdot}00}2704 = 52^2 \\
\hline
& \phantom{= {\cdot}0}39764 \\
& 69587 \\
& 19882 \\
\hline
\textit{Area required} & = 26{\cdot}880464 \text{ sq. inches.}
\end{array}
$$

4. The versed sine is 5, and the diameter 25, find the are of the segment.

Here $\frac{5}{25} = \frac{1}{5} = \cdot 2$.

Now tabular area corresp. to $\cdot 2 = \cdot 111823$

$$\begin{array}{r} 625 = 25^2 \\ \hline 559115 \\ 223646 \\ 670938 \\ \hline \end{array}$$

Area required $= 69\cdot 889375$

5. What is the area of the segment whose height is 3 fee and the diameter of the circle 8 feet?

Here $\frac{3}{8} = \cdot 375$

Tabular area to $\cdot 375 = \cdot 269013$

$$\begin{array}{r} 64 = 8^2 \\ \hline 1076052 \\ 1614078 \\ \hline \end{array}$$

Area required $= 17\cdot 216832$

6. The height or versed sine of a segment is 9 inches, an the diameter $3\frac{1}{6}$ feet, find the area.

$3\frac{1}{6}$ feet $= 38$ inches.

Also $\frac{9}{38} = \cdot 236\,\frac{16}{19}$

Now Tabular area to $\cdot 236 = \cdot 141537$

Do. to $\cdot 237 = \cdot 142387$

$$\begin{array}{r} \cdot 000850 \\ 16 \\ \hline 5100 \\ 850 \\ \hline 19)\cdot 013600 \end{array}$$

Part corresp. to $\frac{16}{19} = \cdot 000715$

Tab. area corresp. to $\cdot 236 = \cdot 141537$

Tab. area corresp. to $\cdot 236\,\frac{16}{19} = \cdot 142252$

$$\begin{array}{r} 1444 = 38^2 \\ \hline 569008 \\ 569008 \\ 569008 \\ 142252 \\ \hline \end{array}$$

Area required $= 205\cdot 411888$ sq. inches

PROBLEM XVII.

To find the area of a circular zone, or of the space included between any two parallel chords, and their intercepted arcs.

Ex. 2. The two parallel chords of a circular zone are 16 and 12, and their perpendicular distance is 2; find the area of the zone.

$$GH = \left\{ PQ^2 + \tfrac{1}{2}(AB^2 + DC^2) + \left(\frac{AB^2 - DC^2}{4PQ}\right)^2 \right\}^{\frac{1}{2}}$$

Here $AB=16$, $DC=12$, $PQ=2$.

$\therefore$ by substitution $GH = \{4 + 200 + 14^2\}^{\frac{1}{2}} = (400)^{\frac{1}{2}} = 20 =$ the diameter of the circle.

Also $mn = \frac{1}{2}GH - \frac{1}{2}\left\{ \left(\frac{AB+DC}{2}\right)^2 + \left(\frac{AB^2 - DC^2}{4PQ}\right)^2 \right\}^{\frac{1}{2}}$

$= 10 - \frac{1}{2}\{14^2 + 14^2\}^{\frac{1}{2}} = 10 - \frac{1}{2}\sqrt{392} = \cdot 1$, very nearly.

Hence $\frac{\cdot 1}{20} = \cdot 005 =$ tabular versed sine.

Now tabular area corresp. to ·005 = ·000470

400 = 20^2

·188000

2

Areas of the two circular segments = ·376000

$(DC + AB) \cdot \frac{PQ}{2} =$ quad. fig. ABCD = 28·

$\therefore$ area of the zone $ABpCDmA$ = 28·376

3. The two parallel sides of a circular zone are 96 and 60, and the breadth 26; what is the area of the zone?

$$\text{Now } GH = \left\{ PQ^2 + \tfrac{1}{2}(AB^2 + DC^2) + \left(\frac{AB^2 - DC^2}{4PQ}\right)^2 \right\}^{\frac{1}{2}}$$

Here $AB=96$, $CD=60$, $PQ=26$.

$\therefore$ *by substitution,* $GH = \{26^2 + 6408 + 54^2\}^{\frac{1}{2}} = (10000)^{\frac{1}{2}} =$ 100 = diameter of the circle.

$$\imath = \tfrac{1}{2}\text{G H} - \tfrac{1}{2}\left\{\left(\frac{\text{A B}+\text{D C}}{2}\right)^2 + \left(\frac{\text{A B}^2-\text{D C}^2}{4\,\text{P Q}}\right)^2\right\}^{\frac{1}{2}} =$$

$3^2+54^2\}^{\frac{1}{2}} = 50 - \frac{1}{2}\sqrt{9000} = 50 - 47\cdot43 = 2\cdot57$

$\cdot025\frac{7}{100}$ = tabular area.

lar area to	·025 =	·005230
	·026	·005546
		·000316
		7
	100)	·002212
		·000022
		·005230
		·005252
		10000 = 100
		52·52
		2
e two segments	=	105·04
ıad. A B C D	=	2028·
ne ABpCDmA	=	2133·04

' the square root of 9000 be taken one decimal place farther, . the tabular area = ·005438, and the area of the zone = 2136·7, ensuration.

ıe two parallel chords of a circular zone are 20 and eir perpendicular distance 17·5; what is the area of

$$= \left\{\text{P Q}^2 + \tfrac{1}{2}(\text{A B}^2+\text{D C}^2) + \left(\frac{\text{A B}^2-\text{D C}^2}{4\,\text{P Q}}\right)^2\right\}^{\frac{1}{2}}$$

= 20, D C = 15, and P Q = 17·5.

$=\{306\cdot25+312\cdot5+6\cdot25\}^{\frac{1}{2}}=\sqrt{625} = 25$ = the dia-
e *circle.*

Also $mn = \frac{1}{2}\,GH - \frac{1}{2}\left\{\left(\frac{AB+DC}{2}\right)^2 + \left(\frac{AB^2 - DC^2}{4\,PQ}\right)^2\right.$

$12{\cdot}5 - \frac{1}{2}\,\{(17{\cdot}5)^2 + 6{\cdot}25\}^{\frac{1}{2}} = 12{\cdot}5 - \frac{1}{2}\sqrt{312{\cdot}5} = 12$
of $17{\cdot}68 = 3{\cdot}66$.

Hence $\frac{3{\cdot}66}{25} = {\cdot}146\frac{2}{5}$ = tabular area.

Now tabular area to $\cdot146 = \cdot071033$
Do. to $\cdot147 = \cdot071741$

Difference $= \cdot000708$
2

$5)\cdot001416$

Part corresp. to $\frac{2}{5} = 000283$
Tab. area to $\cdot146 = \cdot071033$

Tab. area corresp. to $\cdot146\frac{2}{5} = \cdot071316$
$625 = 25^2$

356580
142632
427896

44·572500
2

Area of the two segments $= 89{\cdot}145000$
Area of the quad. ABCD $= 306{\cdot}25$

Area required $= 395{\cdot}395$

5. Find the area of a circular zone each of whose p
chords is 40 feet; the breadth of the zone being 36 feet

$$GH = \left\{PQ^2 + \tfrac{1}{2}(AB^2 + DC^2) + \left(\frac{AB^2 - DC^2}{4\,PQ}\right)^2\right\}^{\frac{1}{2}}$$

Here $AB = CD = 40$, and $PQ = 36$.

$\therefore GH = \{36^2 + 1600\}^{\frac{1}{2}} = \sqrt{2896} = 53{\cdot}81$ = the dia

Also $mn = \frac{1}{2}\,GH - \frac{1}{2} \times \sqrt{40^2} = 26{\cdot}9 - 20 = 6{\cdot}9$

vers. sine. Also $\frac{6{\cdot}9}{53{\cdot}81} = \cdot128\frac{1}{4}$ nearly.

Now tabular area to ·128 = ·058658
Do. to ·129 = ·059327

Difference = ·000669
4)
·000167
·058658

Tab. area corresp. to $128\frac{1}{4}$ = ·058825
2896 = (diameter)^2

352950
529425
470600
117650

170·3572
2

340·7144 = area of two segments.
1440· = area of quad. A B C D.

1780·7144 = area required.

6. Determine the area of a circular zone, one side of which passes through the centre of the circle, and is 30 feet in length; the lesser side being 16 feet.

In this case, since one of the sides passes through the centre, this side will be the diameter of the circumscribing circle = 30 feet.

$$\text{Also } mn = \tfrac{1}{2}\,\text{G H} - \tfrac{1}{2}\left\{\left(\frac{\text{A B}+\text{D C}}{2}\right)^2 + \left(\frac{\text{A B}^2 - \text{D C}^2}{4\,\text{P Q}}\right)^2\right\}^{\frac{1}{2}}$$

$$\text{Also P Q} = (15^2 - 8^2)^{\frac{1}{2}} = \sqrt{161} = 12{\cdot}68.$$

$$\textit{herefore} = 15 - \tfrac{1}{2}\,\{23^2 + (12{\cdot}7)^2\}^{\frac{1}{2}} = 15 - 13{\cdot}135 = 1{\cdot}8$$

Hence $\frac{1{\cdot}865}{30} = {\cdot}062\frac{1}{6}$ = tabular versed sine.

Tabular area corresp. to ·062 = ·020196
Do. to ·063 = ·020680

Difference = ·000484
6)
·000080
·020196

Tab. area corresp. to ·062⅓ = ·020276
900

18·2484
2

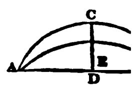
Area of the two segments = 36·4968
Area of the quadrilateral = 291·64

Area of the zone = 328·1368

PROBLEM XVIII.

To find the area of a lune, or of the space included betwee intersecting arcs of two eccentric circles.

Ex. 2. If the length of the chord A B be 9 feet, an height of the segments 4 and 2 feet respectively, determi area of the lune.



Let Δ = diameter of the circle A E B.
δ = diameter of the circle A C B.

$$\left.\begin{array}{l}\text{Then } \Delta \cdot ED = ED^2 + DA^2 \\ \delta \cdot CD = CD^2 + DA^2\end{array}\right\} \text{Euclid. B. 6. Prop. 8. Cor.}$$

Hence $\Delta \times 2 = 2^2 + (4{\cdot}5)^2 = 24{\cdot}25 \therefore \Delta = 12{\cdot}12$
Also $\delta \times 4 = 4^2 + (4{\cdot}5)^2 = 36{\cdot}25 \therefore \delta = 9{\cdot}06$

$$\frac{4}{9{\cdot}0625} = {\cdot}441\tfrac{1}{3} \text{ nearly.}$$

$$\frac{2}{12{\cdot}125} = {\cdot}165 \text{ nearly.}$$

Tab. area corresp. to $\cdot 441\frac{1}{3} = \cdot 334167$

$82\cdot 128 = (9\cdot 0625)^2$ nearly.

$$\begin{array}{r} 2673336 \\ 668334 \\ 334167 \\ 668334 \\ 2673336 \\ \hline \end{array}$$

Area of segment A C B $= 27\cdot 444467376$

Tab. area to $\cdot 165 = \cdot 084801$

$146\cdot 9 = (12\cdot 125)^2$ nearly.

$$\begin{array}{r} 763209 \\ 508806 \\ 339204 \\ 84801 \\ \hline \end{array}$$

Area of segment A E B $= 12\cdot 4572669$
Area of segment A C B $= 27\cdot 4446737$

Area of lune A C B E A $= 14\cdot 9874068$

3. The chord is 20, and the heights of the segments 10 and 2: required the area of the lune.

Here $\Delta \cdot \text{ED} = \text{ED}^2 + \text{DA}^2$

$\delta \cdot \text{CD} = \text{CD}^2 + \text{DA}^2$

But ED $= 2$, CD $= 10$, and DA $= 10$; $\therefore$ by substitution,

$2\,\Delta = 104$, and $\Delta = 52$.

Also $10\ \delta = 200$; $\therefore\ \delta = 20$.

Likewise $\frac{2}{52} = \cdot 038\frac{6}{13}$ = tab. vers. to segment A E B.

And $\frac{10}{20} = \cdot 5$ = tab. vers. to segment A C B, which is a semicircle.

Now tab. area corresp. to $\cdot 038\frac{6}{13} = \cdot 009940$

$2704 = 52^2$

$$\begin{array}{r} 39760 \\ 69580 \\ 19880 \\ \hline \end{array}$$

Area of segment A E B $= 26\cdot 877760$
Similarly, area of segment A C B $= 157\cdot 079600$
Area of lune A E B C $= 130\cdot 201840$

4. The length of the chord is 48, and the heights of the segments 18 and 7: what is the area of the lune?

$$\begin{aligned} \text{Here } \Delta \cdot \text{ED} &= \text{ED}^2 + \text{DA}^2 \\ \text{or } 7\,\Delta &= 49 + 576 = 625 \\ \therefore \Delta &= \tfrac{625}{7} = 89{\cdot}285 \\ \text{Also } 18\,\delta &= 18^2 + 24^2 = 900 \\ \therefore \delta &= 50 \end{aligned}$$

Now $\frac{7}{89{\cdot}285} = {\cdot}078\frac{7}{17}$ nearly = tab. vers. to segment A E B
$\frac{18}{50} = {\cdot}36$ = tab. vers. to segment A C B.
Now Tab. area corresp. to $\cdot36 = {\cdot}254550$

$$\begin{array}{r} 2500 = 50^2 \\ \hline 127275 \\ 50910 \\ \hline \end{array}$$

Area of segment A C B = $636{\cdot}375000$
Similarly, Area of segment A E B = $227{\cdot}77$

$\therefore$ Area of the lune A E B C = $408{\cdot}605$

5. The length of the chord is 15, and the heights of the segments 7 and 4: what is the area of the lune?

$$\begin{aligned} \text{Here } \Delta \cdot \text{ED} &= \text{ED}^2 + \text{DA}^2 \\ \delta \cdot \text{CD} &= \text{CD}^2 + \text{DA}^2 \end{aligned}$$

$$\begin{aligned} \text{That is, } 4\Delta &= 16 + (7{\cdot}5)^2 = 72{\cdot}25 \\ \text{and } 7\delta &= 49 + (7{\cdot}5)^2 = 105{\cdot}25 \end{aligned}$$

Hence $\Delta = 18{\cdot}06$ = diameter of circle A E B.
and $\delta = 15{\cdot}03$ = diameter of circle A C B.

$\frac{4}{\quad} = {\cdot}221\frac{1}{2}$ = Tabular vers. to segment A E B, nearly.
${\cdot}465\frac{2}{3}$ = Tabular vers. to segment A C B, nearly.

Tab. area corr. to ·465 = ·357727
Ditto ·466 = ·358725

·000998
2

3)·001996

·000665
·357727

lar area corr. to $·465\frac{2}{3}$ = ·358392
225·9 = (diameter)^2 nearly

3225528
1791960
716784
716784

a of segment A C B = 80·9607528
rder to find the area of segment A E B, we have
Tab. area corr. to ·221 = ·128942
Ditto ·222 = ·129773

·000831
2)

·000415
·128942

ar area corr. to $·221\frac{1}{2}$ = ·129357
326·16 = (diameter)^2 nearly.

776142
129357
776142
258714
388071

of segment A E B = 42·19107912
80·9607528

of lune A E B C = 38·76967368

6. If the length of a chord be $8\frac{1}{2}$ inches, and the hei versed sines of two circular segments be $5\frac{1}{2}$ and $3\frac{1}{2}$ incl spectively; determine the area of the lune.

$$\text{Here } \Delta . \text{ E D} = \text{E D}^2 + \text{D A}^2$$
$$\delta . \text{ C D} = \text{C D}^2 + \text{D A}^2$$
$$\text{That is, } 3{\cdot}5\ \Delta = (3{\cdot}5)^2 + (4{\cdot}25)^2 = 30{\cdot}3125$$
$$\text{and } 3{\cdot}5\ \delta = (5{\cdot}5)^2 + (4{\cdot}25)^2 = 48{\cdot}3125$$
$$\therefore \Delta = 8{\cdot}66 \text{ nearly; and } \delta = 8{\cdot}784, \text{ nearly.}$$

Now, since the greater diameter $= 8{\cdot}784$, and the of its segment $= 5{\cdot}5$, which is greater than radius; th the area of the segment is greater than that of a sem and is really equal to the area of the circle, whose diam $8{\cdot}784$, *minus* the area of a segment whose versed sine is $3{\cdot}284$

$$\text{Now } \frac{3{\cdot}284}{8{\cdot}784} = {\cdot}373\tfrac{7}{8} \text{ nearly.}$$

Now Tab. area corr. to $\cdot373 = \cdot267078$
Ditto corr. to $\cdot374 = \cdot268045$

$$\begin{array}{r}
{\cdot}000967 \\
7 \\
\hline
{\cdot}006769 \\
8)\overline{\phantom{{\cdot}000846}} \\
{\cdot}000846 \\
{\cdot}267078 \\
\hline
\end{array}$$

Tab. area corr. to $\cdot373\frac{7}{8} = \cdot267924$
$77{\cdot}158 = (8{\cdot}784)^2$ nea

$$\begin{array}{r}
2143392 \\
1339620 \\
267924 \\
1875468 \\
1875468 \\
\hline
\end{array}$$

Area of the segment whose vers. is $3{\cdot}284$ $\Big\} = 20{\cdot}672479992$

$60{\cdot}5998932$ = area of whole (

$39{\cdot}927413208$ = area of seg.

To find the area of the lesser segment A E B.

we have $\frac{3\cdot5}{8\cdot66} = \cdot404\frac{1}{8}$ nearly.

$$
\begin{array}{lrl}
\text{Now tabular area} & \text{to } \cdot404 = & \cdot297292 \\
\text{Do.} & \text{to } \cdot405 = & \cdot298273 \\
\hline
 & & \cdot000981 \\
 & 8) & \overline{\cdot000122} \\
 & & \cdot297292 \\
\hline
\text{Tabular area to } \cdot404\frac{1}{8} = & & \cdot297414 \\
 & & 75 = (8\cdot66)^2 \text{ nearly.} \\
\hline
 & & 1487070 \\
 & & 2081898 \\
\hline
\text{Area of segment A E B} = & & 22\cdot306050 \\
\text{Area of segment A C B} = & & 39\cdot927413 \\
\hline
\therefore \text{Area of lune A E B C} = & & 17\cdot621363 \text{ sq. inches.}
\end{array}
$$

THE CONIC SECTIONS.

PROBLEM II.

In an ellipsis, any three of the four following quantities being ven; viz. the transverse and conjugate diameters, an ordinate d its abscissa, to find the fourth.

CASE I.—*When the transverse, conjugate, and abscissa are ven, to find the ordinate.*

Ex. 2. Given the transverse A B = 35, the conjugate C D = , and the abscissa A N = 28; to find the ordinate.

The transverse = 35

One abscissa = 28

$\therefore$ the other abscissa = 7; hence by the rule

$35 : 25 :: \sqrt{28 \times 7}$: ordinate

or $7 : 5 :: 14$: ordinate $= \frac{70}{7} = 10$, as required.

3. Given the transverse diameter A B = 45, the conjugate C D = 25; and the greater abscissa = 30: find the ordinate.

Transverse = 45
Greater abscissa = 30
Lesser abscissa = 15; hence by the rule,

$45 : 25 :: \sqrt{30 \times 15}$: ordinate

Or $9 : 5 :: 21{\cdot}213$: ordinate $= \frac{106\cdot065}{9} = 11{\cdot}785$ as required.

Case II.—*Having given the transverse, conjugate, and ordinate, to determine the abscissa.*

Ex. 2. The major and minor axes being 35 and 25, what are the abscissæ, when the ordinate is 10^2?

As $25 : 35 :: (12{\cdot}5^2 - 10^2)^{\frac{1}{2}}$: dist. between ord, and centre.
Or $25 : 35 :: 6{\cdot}5$: distance O N = 10·5
But A O = 17·5 = semitransv.
$\therefore$ A O + O N = 28 = greater abscissa.
Also A O — O N = 7 = lesser abscissa.

3. The major and minor axes being 36 and 24, what are the abscissæ, when the ordinate is 8?

$24 : 36 :: (12^2 - 8^2)^{\frac{1}{2}}$: dist. between ord. and centre.
$2 : 3 :: \sqrt{80}$: dist. between ord. and centre.
$\therefore$ distance between ord. and centre $= 6\sqrt{5}$
semi-transverse = 18
$\therefore 18 + 6\sqrt{5} = 6\{3 + \sqrt{5}\} =$ greater abscissa.
$18 - 6\sqrt{5} = 6\{3 - \sqrt{5}\} =$ lesser abscissa.

Case III.—*Having given the conjugate, ordinate, and abscissa, to find the transverse.*

Ex. 2. The conjugate diameter is 25, the ordinate is 10 and the lesser abscissa is 7; what is the length of the transverse diameter?

Semi-conjugate = 12·5
$12{\cdot}5 + (12{\cdot}5^2 - 10^2)^{\frac{1}{2}} = 12{\cdot}5 + 7{\cdot}5 = 20$
Then as $10^2 : 25 \times 7 :: 20$: transverse diameter.
$\therefore$ transverse = 35 as required.

3. *The* conjugate is 30, the ordinate is 9, and the lesser a *is 12:* find the transverse diameter.

Semi-conjugate $= 15$

Hence $15 + (15^2 - 9^2)^2 = 15 + 12 = 27$

Then as $9^2 : 30 \times 12 :: 27$: transverse diameter.

Or $9 : 360 :: 3$: transverse diameter.

$\therefore$ the transverse $= 120$ as required.

Case IV.—*Having given the transverse, ordinate, and abscissa, to find the conjugate.*

Ex. 2. The transverse diameter is 35, the ordinate 10, and the lesser abscissa $= 7$; find the conjugate.

Transverse $= 35$

Lesser abscissa $= 7$; $\therefore$ greater abscissa $= 28$

$\therefore \sqrt{28 \times 7} : 10 :: 35$: conjugate

Or $14 : 10 :: 35$: conjugate

$\therefore$ conjugate $= \frac{350}{14} = 25.$

3. The transverse diameter is 48, the ordinate 16, and its lesser abscissa 12; find the conjugate.

The transverse $= 48$

Lesser abscissa $= 12$

$\therefore$ Greater abscissa $= 36$.

Hence $\sqrt{36 \times 12} : 16 :: 48$: conjugate

Or $12\sqrt{3} : 16 :: 48$: conjugate

$$\therefore \text{conjugate} = \frac{64}{\sqrt{3}}$$

PROBLEM III.

To find the circumference of an ellipse, the transverse and conjugate diameters being known.

Ex. 2. The transverse and conjugate are 24 and 20 respectively; find the periphery or circumference of the ellipse.

Here, $\left\{\frac{24^2+20^2}{2}\right\}^{\frac{1}{2}} = \sqrt{488} = 22{\cdot}09$ nearly.

$$\begin{array}{r} 22{\cdot}09 \\ 3{\cdot}1416 \\ \hline 282744 \\ 62832 \\ 62832 \\ \hline 69{\cdot}397944 \end{array} = \text{circumference of the ellipse.}$$

3. The two axes of an ellipse are 25 and 15 ; wha length of the circumference?

$$\left\{\frac{25^2+15^2}{2}\right\}^{\frac{1}{2}} = \sqrt{425} = 20{\cdot}61$$

$$\begin{array}{r} 20{\cdot}61 \\ 3{\cdot}1416 \\ \hline 31416 \\ 188496 \\ 62832 \\ \hline 64{\cdot}748376 \end{array} = \text{circumference of the}$$

PROBLEM IV.

To determine the area of an ellipse, the transverse a jugate diameters being given.

Ex. 2. If the axes of an ellipse be 35 and 25, what area?

$$\begin{array}{r} 35 \\ 25 \\ \hline 175 \\ 70 \\ \hline 875 \\ {\cdot}7854 \\ \hline 39270 \\ 54978 \\ 62832 \\ \hline 687{\cdot}225 \end{array} = \text{area.}$$

3. Required the area of an ellipsis whose two axes and 50.

$$\begin{array}{r} 70 \\ 50 \\ \hline 3500 \\ .7854 \\ \hline 39270 \\ 23562 \\ \hline \end{array}$$

area of the ellipse = 2748.9000

'he major axis of the ellipse in Grosvenor square is 840 and the minor axis 612 links, required the number of which the inclosure contains.

```
          612
           840
        -------
         24480
        4896
        -------
        514080
          ·7854
        --------
        2056320
       257040
      411264
     359856
     ----------
10000)403758·4320 sq. links
     ------------
   10)40·3758432  sq. chains
     ------------
      4·03758432 acres
               4
     ------------
       ·15033728 roods
              40
     ------------
      6·01349120 perches.
     ------------
```

Answer—4 acres, 0 roods, 6 perches.

PROBLEM V.

ind the area of an elliptic segment, whose base is parallel er axis of the ellipse.

2. What is the area of an elliptic segment cut off by a : ordinate parallel to the conjugate axis, at the distance rom the centre, the axes being 120 and 40?

distance of the base of the segment from the vertex = 6 = 24.

$\therefore \frac{24}{120} = \frac{1}{5} = \cdot 2 =$ the tabular versed sine.

Now tabular area to ·2 = ·111823
120

13·418760
40

Area required = 536·750400

3. Determine the area of an elliptic segment whose a is 8 inches; the two axes of the ellipse being 4 feet and also the chord of the segment being supposed parallel major axis.

Here $\frac{8}{36} = \frac{2}{9} = \cdot 222\frac{2}{9}$ = tabular versed sine.

Now tabular area to ·222 = ·129773
Do. to ·223 = ·130605

·000832
2

9)·001664

·000185 nearly.
·129773

Tabular area to ·222 $\frac{2}{9}$ = ·129958
48 = major axis.

1039664
519832

6·237984
36 = minor axis.

37427904
18713952

Area = 224·567424 sq. inches.

4. What is the area of a segment, cut off by an ordinate parallel to the transverse diameter, whose height is 10, the axes being 35 and 25.

$$\frac{10}{25} = \frac{2}{5} = \cdot 4 = \text{tabular versed sine.}$$

Now, tab. area answering to ·4 = ·293369

$$\begin{array}{r} \cdot 293369 \\ 35 \\ \hline 1466845 \\ 880107 \\ \hline 10\cdot 267915 \\ 25 \\ \hline 51339575 \\ 20535830 \\ \hline \end{array}$$

Area required = 256·697875

PROBLEM VII.

In a parabola, any three of the four following quantities being given, viz. any two ordinates and their two abscissæ, to find the fourth.

Ex. 2. The ordinate G H is 8, and its abscissa A H is 16; find the abscissa A N, whose ordinate E N is 6.

As A H : G H^2 : : abscissa required A N : E N^2

or 16 : 64 : : abs. req. : 36.

$\therefore$ Abscissa required = 9.

3. The ordinate E N is 8, and its abscissa A N is 10, find the ordinate G H, whose abscissa A H is 22.

$10 : 8^2 : : 22 : (\text{ord. req.})^2$

$\therefore$ Ordinate required $= 8 \cdot (\frac{22}{10})^{\frac{1}{2}} = 8\sqrt{\frac{11}{5}}$.

PROBLEM VIII.

To find the length of any arc of a parabola, cut off by a double ordinate.

Ex. 2. The abscissa A H being equal to 3, and its ordinate G H equal to $4\frac{1}{2}$; *what is the length* of the arc G A K?

$(4\cdot5)^2 + \frac{4}{3} \times 3^2 = 32\cdot25$ (5·678

```
              32·25  (5·678
              25        2
             ----    ------
      106)   725     11·356 = arc G A K nearly.
             636     ------
             -----
     1127)   8900
             7889
             -----
    11348)101100
           90784
           ------
           10316
           ------
```

3. The abscissa A H being equal to 8, and the ordinat of the same length; what is the length of the arc G A K?

$8^2 + \frac{4}{3} \times 8^2 = 64 + 85\cdot333 = 149\cdot3333$, &c.

```
            149·3333 (12·22
            1           2
            ----      -----
      22)   49        24·44 = arc G A K, nearly.
            44        -----
            -----
     242)   533
            484
            -----
    2442)   4933
            4884
            ----
              49
            ----
```

4. If the abscissa A H $= 8\frac{1}{2}$ inches, the ordinate G H = inches; determine the length of the arc G A K.

$(\frac{27}{4})^2 + \frac{4}{3}(\frac{17}{2})^2 = 45\cdot5625 + 96\cdot3333 = 141\cdot895$

```
      141·8958  (11·91
      1              2
      ———          ———
21)   41         23·82 = arc G A K, nearly.
      21          ———
      ———
229)  2089
      2061
      ———
2381)  2858
       2381
       ———
        477
       ———
```

PROBLEM IX.

d the area of a parabola, its base or double ordinate and altitude being given.

2. The abscissa A N = 12, and the double ordinate or 9; what is the area?

```
  39
  12
 ———
 468
   2
 ———
3)936
 ———
 312 = area of the parabola.
 ———
```

What is the area of a parabola, whose abscissa A H is 10, dinate G H = 8?

```
 16
 10
 ———
 160
   2
 ———
3)320
 ———
 106⅔ = area of the parabola.
 ———
```

D

4. Find the area of a parabola whose abscissa A H is and ordinate G H 6·75.

$$
\begin{array}{r}
\text{Double ordinate} = 13.50 \\
11{\cdot}25 \\
\hline
56250 \\
3375 \\
1125 \\
\hline
151{\cdot}8750 \\
2 \\
\hline
3)303{\cdot}7500 \\
\hline
101{\cdot}25 = \text{area of the pa}
\end{array}
$$

PROBLEM X.

To find the area of the frustum of a parabola.

Ex. 2. The greater end of a parabolic frustum is lesser end 20, and their distance 6: what is the area?

$$25^3-20^3 = 7626$$
$$25^2-20^2 = 225$$
$$\therefore \tfrac{7625}{225} = 33{\cdot}888$$
$$\underline{4} = \tfrac{2}{3} \text{ of altitude.}$$
$$135{\cdot}552 = \text{area required.}$$

3. Required the area of a parabolic frustum, the grea of which is 10, and the lesser 6; their distance being 9

$$10^3-6^3 = 784$$
$$10^2-6^2 = 64$$
$$\text{and } \tfrac{784}{64} = \tfrac{196}{16}$$
$$\text{Hence } \tfrac{196}{16} \times \tfrac{2}{3} \times 9 = \tfrac{196}{16} \times 6 = \tfrac{588}{8} =$$

4. Find the area of the frustum of a parabola, the tw being 9 and 15, and their distance 8 inches.

$$15^3-9^3 = 2646$$
$$15^2-9^2 = 144$$
$$\text{and } \tfrac{2646}{144} = \tfrac{147}{8}$$

Hence $\tfrac{147}{8} \times \tfrac{16}{3} = \tfrac{294}{3} = 98$ sq. inches = the area lic frustum as required.

PROBLEM XII.

In an hyperbola, any three of the four following quantities ng given, viz. the transverse and conjugate diameters, an or- ate and its abscissa, to find the fourth.

CASE I.—*Having given the transverse and conjugate diameters, l the two abscissæ, to find the ordinate.*

Ex. 2. If the transverse diameter be 24, the conjugate 16, d the lesser abscissa 8; what is the ordinate?

As $24 : 16 :: (8 \times 32)^{\frac{1}{2}}$: ordinate;
or $3 : 2 :: 16$: ordinate;
$\therefore$ ordinate required $= \frac{32}{3} = 10\frac{2}{3}$.

3. The transverse diameter or major axis of an hyperbola is , the conjugate or minor axis 40, and the greater abscissa ; required the ordinate.

$64 - 50 = 14 =$ lesser abscissa.
$\therefore 50 : 40 :: (64 \times 14)^{\frac{1}{2}}$: ordinate.
or, $5 : 4 :: 8\sqrt{14}$: ordinate;
$\therefore$ ordinate $= \frac{32}{5}\sqrt{14}$.

4. The transverse diameter of an hyperbola is 45, the con- gate 15, and the lesser abscissa 4; required the ordinate.

$45 + 4 = 49 =$ greater abscissa.
Hence $45 : 15 :: (4 \times 49)^{\frac{1}{2}}$: ordinate;
or, $3 : 1 :: 14$: ordinate;
$\therefore$ ordinate $= \frac{14}{3} = 4 \cdot 666$.

CASE II.—*Having given the major and minor axes, and an dinate, to determine the abscissæ.*

Ex. 2. The transverse and conjugate diameters are 24 and ; required the two abscissæ to the ordinate 14.

$$(\text{ordinate})^2 = 196$$
$$(\text{semi minor})^2 = \tfrac{441}{4}$$
$$\therefore \text{ sum of these} = \frac{784 + 441}{4} = \frac{1225.}{4}$$

Hence $21 : 24 :: (\frac{1225}{4})^{\frac{1}{2}}$: distance between ord. and centr
or $7 : 8 :: \frac{35}{2}$: distance between ord. and centre.
this distance $= \frac{140}{7} = 20.$
Hence $20 + 12 = 32$ = greater abscissa.
$20 - 12 = 8$ = lesser abscissa.

3. The transverse being 60, and the conjugate 36; requir the two abscissæ to the ordinate 24.

$$(\text{semi-conj.})^2 = 18^2 = 324$$
$$(\text{ordinate})^2 = 24^2 = 576$$
$$\text{Sum of these} = 900$$
$$\text{whose square root} = 30$$

Hence $36 : 60 :: 30 : \frac{1800}{36} = 50$ = dist. between ord. and cent
$\therefore 50 + 30 = 80$ = greater abscissa.
$50 - 30 = 20$ = lesser abscissa.

4. The transverse diameter being 50, the conjugate 30, a the ordinate 6, find the two abscissæ.

$$(\text{semi conjugate})^2 = 15^2 = 225$$
$$(\text{ordinate})^2 = 6^2 = 36$$
$$\text{Sum of these} = 261$$
$$\text{whose square root} = \sqrt{261} = 3\sqrt{29}.$$

Hence $30 : 50 :: 3\sqrt{29}$: dist. betw. ord. and centre;
or $3 : 5 :: 3\sqrt{29}$: dist. betw. ord. and centre.
This distance $= 5\sqrt{29}$
Hence $5\sqrt{29} + 25 = 5\{\sqrt{29} + 5\}$ = greater abscis
and $5\sqrt{29} - 25 = 5\{\sqrt{29} = 5\}$ = lesser abscissa

CASE III.—*Having given the major axis, the two abscis and the ordinate, to determine the minor axis.*

Ex. 2. The transverse diameter is 24, the ordinate 16, a the greater abscissa 32; determine the conjugate.

$32 - 24 = 8 =$ lesser abscissa.

Hence $(8 \times 32)^{\frac{1}{2}} : 16 :: 24 :$ conjugate.

or $16 : 16 :: 24 :$ conjugate.

$\therefore$ conjugate axis $= 24$, which also $=$ transverse.

3. The transverse diameter is 60, the ordinate 15, and the abscissæ 80 and 20; determine the conjugate.

$80 - 60 = 20 =$ lesser abscissa.

$\therefore (80 \times 20)^{\frac{1}{2}} : 15 :: 60 :$ conjugate.

or $40 : 15 :: 60 :$ conjugate.

$\therefore$ conjugate $= \frac{90}{4} = 22\frac{1}{2}$.

4. The transverse diameter is 24, the ordinate 5, and the abscissæ 30 and 6; determine the conjugate.

As $(30 \times 6)^{\frac{1}{2}} : 5 :: 24 :$ conjugate.

or $6\sqrt{5} : 5 :: 24 :$ conjugate.

$\therefore$ conjugate $= 4\sqrt{5}$.

Case IV.—*Having given the minor axis, the ordinate, and the two abscissæ, to find the major axis.*

Ex. 2. The conjugate diameter $= 16$, the ordinate $= 10\frac{2}{3}$, and the greater abscissa $= 32$; find the value of the transverse diameter.

$$\text{semi conj.} = 8$$
$$\text{ordinate} = \tfrac{32}{3}$$
$$\therefore \left(8^2 + \left(\tfrac{32}{3}\right)^2\right)^{\frac{1}{2}} = \left(\tfrac{1600}{9}\right)^{\frac{1}{2}} = \tfrac{40}{3}$$
$$\text{Hence } \frac{40}{3} - 8 = \frac{40-24}{3} = \frac{16}{3}$$
$$\therefore \left(\tfrac{32}{3}\right)^2 : 32 \times 16 :: \tfrac{16}{3} : \text{major axis.}$$
$$\text{or } \tfrac{32}{9} : 16 :: \tfrac{16}{3} : \text{major axis.}$$
$$\therefore \text{major axis} = 24.$$

3. The conjugate diameter is $4\sqrt{5}$, the lesser abscissa 6, and its ordinate 5; required the transverse.

$$\text{semi conj.} = 2\sqrt{5}$$
$$\text{ordinate} = 5$$
$$\therefore (5^2 + 20)^{\frac{1}{2}} = (45)^{\frac{1}{2}}$$
$$\therefore 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}.$$

$$\text{Hence } 5^2 : 6 \times 4\sqrt{5} :: 5\sqrt{5} : \text{major axis.}$$

$$\therefore \text{major axis} = \frac{6 \times 4 \cdot 5^2}{5^2} = 24.$$

4. Required the transverse diameter of the hyperbola wh[o]
conjugate is 36, the lesser abscissa being 20, and ordinate 2[4]

$$\text{semi conj.} = 18$$
$$\text{ordinate} = 24$$
$$\therefore (18^2 + 24^2)^{\frac{1}{2}} = (900)^{\frac{1}{2}} = 30$$
$$\text{therefore } 30 + 18 = 48$$
$$\text{Consequently } 24^2 : 20 + 36 :: 48 : \text{transverse};$$
$$\therefore \text{transverse} = 60.$$

5. Determine the transverse diameter of an hyperbola wh[ose]
conjugate is 24, the ordinate being 8, and the greater [ab-]
scissa 36.

$$\text{semi conj.} = 12$$
$$\text{ordinate} = 8$$
$$(12^2 + 8^2)^{\frac{1}{2}} = (144 + 64)^{\frac{1}{2}} = \sqrt{208}$$
$$\sqrt{208} - 12 = \text{the difference.}$$
$$\text{Then as } 8^2 : 36 \times 24 :: \sqrt{208} - 12 : \text{major axis};$$
$$\text{or } 8 : 108 :: \sqrt{208} - 12 : \text{major axis,}$$
$$\therefore \text{major axis} = \tfrac{27}{2}\left\{\sqrt{208} - 12\right\} = 54\sqrt{13} - 162 = 32{\cdot}67$$

PROBLEM XIII.

To find the length of any arc of an hyperbola, reckoning fro[m] the vertex.

Ex. 2. The transverse diameter of an hyperbola is 1[...]
conjugate 72, the ordinate 48, and the abscissa 40[...]
the length of the curve.

$$
\begin{array}{r}
21 \times 72^2 = 108864 \\
19 \times 120^2 = 273600 \\
\hline
19\,a^2 + 21\,b^2 = 382464 \\
40 \\
\hline
15298560 \\
15\,ab^2 = 9331200 \\
\hline
24629760
\end{array}
$$

$$
\begin{array}{r}
\text{Also } 21 \times 72^2 = 108864 \\
9 \times 120^2 = 129600 \\
\hline
9\,a^2 + 21\,b^2 = 238464 \\
40 \\
\hline
9538560 \\
15 \times 120 \times 72^2 = 9331200 \\
\hline
18869760
\end{array}
$$

$\frac{976}{976} \times 48 = 1{\cdot}305 \times 48 = 62{\cdot}65$ nearly $=$ the length
/perbolic arc AQ.

e transverse diameter and conjugate are respectively
16; required the length of the curve to an ordinate 8.

$$
\begin{array}{r}
21 \times 16^2 = 5376 \\
19 \times 20^2 = 7600 \\
\hline
12976
\end{array}
$$

s case it will be necessary first to find the lesser ab-

case 2, Prob. xii.,
: 20 : : $\sqrt{8^2 + 8^2}$: dist. between ord. and centre,
5 : : $8\sqrt{2}$: dist. between ord. and centre,
istance $= 10\sqrt{2}$.
$\therefore 10\sqrt{2} - 10 = 4{\cdot}142 =$ the abscissa.

$$\begin{array}{rr}
\text{Now } 21 \times 16^2 = & 5376 \\
19 \times 20^2 = & 7600 \\
\hline
& 12976 \\
& 4.142 \\
\hline
& 25952 \\
& 51904 \\
& 12976 \\
& 51904 \\
\hline
& 53746.592 \\
15 \times 20 \times 16^2 = & 76800. \\
\hline
& 130546.592
\end{array}$$

$$\begin{array}{rr}
\text{Also } 21 \times 16^2 = & 5376 \\
9 \times 20^2 = & 3600 \\
\hline
& 8976 \\
& 4.142 = \text{absc} \\
\hline
& 17952 \\
& 35904 \\
& 8976 \\
& 35904 \\
\hline
& 37178.592 \\
15 \times 20 \times 16^2 = & 76800 \\
\hline
& 113978.592
\end{array}$$

$\frac{130546.592}{113978.592} \times 8 = 1.145 \times 8 = 9.16 =$ the length hyperbolic arc A Q; hence $8.164 \times 2 = 16.328 =$ len arc PAQ.

4. Determine the length of an hyperbolic arc, whose or is 15; the transverse and conjugate axes being 80 and 6

In order to determine the lesser abscissa, say

$: 80 : : \sqrt{15^2 + 30^2}$: dist. between ord. and cen

$4 : : \sqrt{1125}$: dist. between ord. and centre.

this distance $= \frac{4}{3}\sqrt{1125} = 54.721$

$\therefore 44.721 - 40 = 4.721 =$ the lesser absc.

$$
\begin{array}{r}
\text{Now } 21 \times 60^2 = 75600 \\
19 \times 80^2 = 121600 \\
\hline
197200 \\
4.721 = \text{abscissa} \\
\hline
197200 \\
3944 \\
13804 \\
7888 \\
\hline
930981{\cdot}200 \\
15 \times 80 \times 60^2 = 4320000{\cdot} \\
\hline
5250981{\cdot}200 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
\text{Also } 21 \times 60^2 = 75600 \\
9 \times 80^2 = 57600 \\
\hline
133200 \\
4.721 \\
\hline
133200 \\
2664 \\
9324 \\
5328 \\
\hline
628837{\cdot}200 \\
15 \times 80 \times 60 = 4320000{\cdot} \\
\hline
4948837.200
\end{array}
$$

[H]ence $\frac{5250981{\cdot}2}{4948837{\cdot}2} \times 15 = 1.061 \times 15 = 15.915 =$ length [of t]he arc AQ; and $15.915 \times 2 = 31{\cdot}83 =$ length of the arc [A]Q.

PROBLEM XIV.

[T]o find the area of an hyperbola; the transverse, conjugate, and abscissa being given

Ex. 2. The transverse diameter is 80, the conjugate 6(the lesser abscissa 30 ; find the area of the hyperbola.

$80 \times 30 = 2400 =$ product of the transverse and abs
$\frac{5}{7}$ of $30^2 =$ 642.857142, &c.

```
          3042.857142(55.162
          25                21
          ----            ------
      105)542             55162
          525            110324
          ------         --------
     1101)1785           1158.402
          1101
          ------
    11026)68471
          66156
          ------
   110322)231542
          220644
          ------
           10898
          ------

          80 × 20 = 2400(48.989
                    16          4
                    ----  ------
                 88)800  195.956
                    704
                    ----
                969)9600
                    8721
                    -----
               9788)87900
                    78304
                    ------
              97969)959600
                    881721
                    ------
                     77879
                    ------
```

```
   1158·402
    195·956
   ---------
75)1354·358(18·058
   75
   ---
   604
   600
   ---
    435
    375
    ---
     608
     600
     ---
       8
```

60 × 30 = 7200 = 4 times the product of the conjugate abscissa.

```
80)——
   90
```

```
Lastly,  18·058
             90
       --------
       1625·220
```

. The transverse *diameter* is 50, the conjugate 30; re-
ed the area of the hyperbola to an abscissa 25.

$50 \times 25 = 1250 =$ prod. of transv. and absciss

$\frac{5}{7} \times 25^2 = 446{\cdot}428571$

```
        1696·428571(41·187
        16               21
     81)  96          41187
          81          82374
    821)1542         864·927
         821
   8228)72185
        65824
  82367)636171
        576569
         59602
```

```
50 × 25 = 1250· (35·3553
          9            4
        65)350  141·4212
           325
       703)2500
           2109
      7065)39100
           35325
     70705)377500
           353525
    707103)2397500
           2121309
            276191
```

```
           864·927
           141·421
         ─────────
        75)1006·348(13·418, nearly.
           75
           ───
           256
           225
           ───
            313
            300
            ───
             134
              75
             ───
             598
             ───
```

× 25 = 3000 = 4 times the prod. of the conjug. and

```
 50)────
     60
     ──
```

Lastly 13·418

60

805·080 = the area, nearly.

ermine the area of an hyperbola, whose major and es are 45 and 30; the abscissa being 10.

$45 \times 10 = 450 =$ prod. of transverse and absc
$\frac{5}{7} \times 10^2 = 71{\cdot}428571$

```
         521·428571(22·8348
         4                   21
        ----            --------
   42)121               228348
       84              456696
      ----             --------
  448)3742             479·5308
      3584             --------
      -----
 4563)15885
      13689
      ------
45664)219671
      182656
      ------
       37015
      ------
```

```
45 × 10 = 450 (21.2132
          4          4
          --    -------
     41)50     84.8528
        41
        ---
    422)900
        844
        ----
   4241)5600
        4241
        -----
  42423)135900
        127269
        ------
 424262)863100
         848524
         ------
          14576
         ------
```

```
 479.5308
  84.8528
 --------
75)564.3836(7.5251
   525
   ---
    393
    375
    ---
     188
     150
     ---
      383
      375
      ---
        86
        75
        --
        11
        --
```

```
4 × 30 × 10 — 1200
           45)------
               26.6666
                7.5251
               -------
                266666
              1333330
              533332
            1333330
           1866662
           ------------
           200.66883166 = area o[f]
           ------------   hyperb[ola]
```

[6]. *Required the area of an hyperbola whose axes ar[e] … the abscissa being 8.*

$30 \times 8 = 240 =$ product of trans. and abscissa.

$\frac{5}{7} \times 8^2 =$ 45.7142857

```
        285.7142857 (16.903
        1                21
       ----           ------
    26)185             16903
        156          33806
       ----          -------
   329)2971          354.963
        2961
       ------
 33803)104285
        101409
       -------
          2876
          ----


    30 × 8 = 240   (15.4919
              1          4
              -   ---------
         25)140   61.9676
             125
            ----
        304)1500
            1216
           ------
       3089)28400
             27801
            -------
      30981)59900
            30981
           --------
     309829)2891900
            2788461
           ---------
             103439
            --------
```

```
   354.963
    61.967
75)416.930(5.559
   375
   ---
    419
    375
    ---
     443
     375
     ---
      680
      675
      ---
        5
```

$4 \times 20 \times 8 = 640 =$ 4 times prod. of conj. and abs.

```
30)-------
    21.3333
      5.559
   ---------
    1919997
   1066665
  1066665
 1066665
 ----------
 118.5918147 = area of the hyperbola.
```

Find the area of an hyperbola whose axes are 36 and 25; *bscissa being 9.*

$36 \times 9 = 324$

$\frac{5}{7}$ of $9^2 = 57{\cdot}8571428$

```
        381·8571428(19·5411
        1                21
       ----          -------
    29)281            195411
        261          390822
       ----          -------
   385)2085          410·3631
        1925         -------
       -----
  3904)16071
        15616
       ------
 39081)45542
        39081
       ------
         6461
       ------

36 × 9 = 324(18
          1   4
          -   -
     28)224  72
        224  —
        ---
   410·3631
     72
75)482·3631(6·4315
   450
   ---
    323
    300
    ---
     236
     225
     ---
      113
       75
      ---
       381
       375
       ---
         6
```

$4 \times 25 \times 9 = 900$

36)——

25

Lastly, 6·4315
25

321575
128630

160·7875 = area of the hyperbola.

PROBLEM XV.

To find the area of a cycloid whose axis is known.

x. 2. The axis of a cycloid is $9\frac{1}{2}$ feet, find the area of the ɔid.

·7854 = area of a circle whose diameter is 1.
90·25 = $(9·5)^2$

39270
15708
70686

70·882350 = area of generating circle.
3

212·647050 = area of the cycloid.

The axis of a cycloid is 5 yards, determine the area of the ›id.

·7854 = area of circle to diameter unity.
25 = 5^2

39270
15708

19·6350 = area of generating circle.
3

58·9050 = area of the cycloid.

4. The base of the cycloid is 25 yards; find the area cycloid.

Since the base of the cycloid is 25 yards, therefore fr nature of the figure, the circumference of the generatin = 25 yards.

$$
\begin{array}{r}
3{\cdot}1416)25{\cdot}0000000(7{\cdot}9577 \\
219912 \\
\hline
300880 \\
282744 \\
\hline
181360 \\
157080 \\
\hline
242800 \\
219912 \\
\hline
228880 \\
219912 \\
\hline
8968
\end{array}
$$

$\therefore$ 7·9577 = the diameter of the generating circl

$$
\begin{array}{rl}
{\cdot}7854 & = \text{area of circle to di} \\
63{\cdot}325 & = (7{\cdot}9577)^2\text{, nearly.} \\
\hline
253300 & \\
316625 & \\
506600 & \\
443275 & \\
\hline
49{\cdot}7354550 & = \text{area of the gen} \\
3 & \quad \text{circle.} \\
\hline
149{\cdot}2063650 & = \text{area of the cycl}
\end{array}
$$

*Otherwise.**

$$
\begin{array}{r}
\cdot 07958 \\
625 \\
\hline
39790 \\
15916 \\
47748 \\
\hline
49\cdot 73750 \\
3 \\
\hline
149\cdot 21250 \\
\hline
\end{array}
$$

$625 = 25^2 = (\text{circumf.})^2$

$49\cdot73750 =$ area of generating circle.

$149\cdot21250 =$ area of the cycloid, nearly as before.

THE MENSURATION OF SOLIDS.

PROBLEM I.

To find the solidity of a cube, the length of one of its sides being given.

Ex. 2. A cube has one of its sides equal to 25·5 inches in length; find its solidity.

$$
\begin{array}{r}
25\cdot 5 \\
25\cdot 5 \\
\hline
1275 \\
1275 \\
510 \\
\hline
650\cdot 25 \\
25\cdot 5 \\
\hline
325125 \\
325125 \\
130050 \\
\hline
\end{array}
$$

Solidity $= 16581\cdot375$ cubic inches.

* See Problem *XIII., Rule II.,* in the Mensuration of Superficies.

3. A side of a cube is 15 inches; find its solidity.

15 inches = 1·25 feet.

```
      1·25
      ----
       625
      250
     125
    ------
    1·5625
      1·25
    ------
     78125
    31250
   15625
   --------
Solidity =1·953125 cubic feet.
```

Otherwise.

```
      15
      15
      --
      75
     15
     ---
     225
      15
    ----
    1125
    225
    ----
1728)3375(1 cub. foot. 11 pts. 5¼ in
     1728
     ----
     1647
       12
    -----
    19764
    19008
    -----
      756
       12
     ----
     9072
     8640
     ----
      432
```

* These are not solid inches, but are $\frac{1}{144}$ ths of a cubic foot; th[ese] *inches*, in reality, contains 12 cubic inches.

Determine the solidity of a cube one of whose sides is 7·5

```
      7·5
      7·5
     ----
     375
    525
    -----
    56·25
      7·5
   ------
   28125
  39375
  -------
```

Solidity = 421·875 cubic feet.

PROBLEM II.

To find the solidity of a solid parallelopiped.

x. 2. The length of a parallelopiped is 9 feet, breadth 2 and depth or thickness 18 inches; determine its solidity.

```
   9 feet = length.
   2 feet = breadth.
  --
  18
  1·5 feet = depth.
  ---
  90
 18
 ---
```

Solidity = 27·0 cubic feet, or 1 cubic yard,

. The length of a parallelopiped is 15 feet, and each side of quare *base* or end is 21 inches; what is the solidity?

21 = side of the sq. end.
21

21
42

441 = area of the end in inche
180 = length of the solid, in i

35280
441

1728)79380 cubic inches.

Solidity = 45·937 cubic feet.

4. What is the solidity of a block of marble, whose is 10 feet, its breadth $5\frac{3}{4}$ feet, and the depth $3\frac{1}{2}$ feet.

10 = length.
5·75 = breadth.

57·5
3·5 = depth.

2875
1725

Solidity = 201·25 cubic feet.

5. How many cubic feet are there in a piece of timb form of a parallelopiped, whose length, breadth, and t are 35 feet, $3\frac{3}{4}$ feet, and $2\frac{1}{2}$ respectively?

35 = length.
3·75 = breadth.

1875
1125

131·25
2·5 = thickness.

65625
26250

= 328·125 cubic feet.

PROBLEM III.

To find the solidity of a prism.

2. Determine the content of a triangular prism, whose is 18 inches, and the length of a side of its equilateral ; 1·5 inches.

1·5
1·5
1·5

2)4·5 = perimeter.

2·25 = ½ perimeter.

a of the base — $\{2{\cdot}25 \times {\cdot}75 \times {\cdot}75 \times {\cdot}75\}^{\frac{1}{2}}$
$= \{{\cdot}94921875\}^{\frac{1}{2}} = {\cdot}97427$
18

779416
97427

Solidity = 17·53686 cubic inches.

Required the solidity of a prism whose base is a hexagon, sing each of the equal sides to be 1 foot 4 inches, and ngth of the prism 15 feet.

Now 2·598076 = area of a hexagon whose side = 1
256 inches = $(\text{a side})^2$ of the prism's base.

15588456
12990380
5196152

144)665·107456 sq. inches = the area of the prism's base.

4·6188 sq. feet = area of prism's base.
15 = length of the prism.

230940
46188

ntent = *69·2820* cubic feet.

4. Find the content of a prism whose base is a regular octagon, a side of which is 3 feet, and the length of the prism 12 feet.

$$\begin{array}{rl} 4{\cdot}828427 = & \text{area of an octagon whose side is [unity.} \\ 9 = & (\text{side})^2 \text{ of prism's base.} \\ \hline 43{\cdot}455843 & \\ 12 = & \text{length of the prism.} \\ \hline \text{Solidity} = 521{\cdot}470116 & \text{cubic feet.} \end{array}$$

5. Find the content of a prism whose base is a regular decagon, a side of which is 2 feet, and the length of the prism 20 feet.

$$\begin{array}{rl} 7{\cdot}694208 = & \text{area of a decagon whose side is [unity.} \\ 4 = & (\text{side})^2 \text{ of prism's base.} \\ \hline 30{\cdot}776832 & \\ 20 & \\ \hline \text{Solidity} = 615{\cdot}536640 & \text{cubic feet.} \end{array}$$

PROBLEM IV.

To find the convex surface of a cylinder.

Ex. 2. What is the convex surface of a right cylinder, the diameter of whose base is 30 inches, and the length of the cylinder 60 inches?

$$\begin{array}{rl} 3{\cdot}1416 = & \text{circumf. of circle to dia. 1.} \\ 30 & \\ \hline 94{\cdot}2480 = & \text{circumf. of cylinder's base.} \\ 60 = & \text{alt. of cylinder.} \\ \hline \text{Convex surface} = 5654{\cdot}8800 & \text{sq. inches.} \end{array}$$

3. Required the convex superficies of a right cylinder whose circumference is 8 feet 4 inches, and its length 7 feet.

$8\frac{1}{3}$ feet = circumf. of cylinder's base.
7

'onvex surface = $58\frac{1}{3}$ = 58·333, &c. sq. feet.

. Find the whole surface of a cylinder, the diameter of whose ; is 5 feet, and its perpendicular altitude 30 feet.

$$
\begin{array}{rl}
3{\cdot}1416 & = \text{circumf. of circle to diameter 1.} \\
5 & \\
\hline
15{\cdot}7080 & = \text{circumf. of cylinder's base.} \\
30 & \\
\hline
471{\cdot}2400 & = \text{convex surface.} \\
\text{Now } {\cdot}7854 & = \text{area of circle to diameter 1.} \\
25 & = (\text{diameter})^2 \text{ of cylinder's base.} \\
\hline
39270 & \\
15708 & \\
\hline
19{\cdot}6350 & = \text{area of one end of the cylinder.} \\
2 & \\
\hline
39{\cdot}2700 & = \text{area of the two ends.} \\
471{\cdot}24 & = \text{area of convex surface.} \\
\hline
\text{Total area} = 510.51 & \text{sq. feet.}
\end{array}
$$

PROBLEM V.

To find the solidity of a cylinder.

x. 2. What is the solidity of a cylinder whose height is 10 and the diameter of the base 2 feet?

$$
\begin{array}{rl}
{\cdot}7854 & = \text{area of circle to diameter 1.} \\
4 & \\
\hline
3{\cdot}1416 & = \text{area of cylinder's base.} \\
10 & = \text{altitude.} \\
\hline
\textit{Solidity} = 31{\cdot}4160 & \text{cubic feet.}
\end{array}
$$

3. What is the solidity of a cylinder whose height is 15 fe
and the circumference of its base 20 feet?

$$
\begin{array}{r l}
\cdot 07958 & \\
400 & = 20^2 = (\text{circumf.})^2 \\
\hline
31.83200 & = \text{area of cylinder's base.} \\
15 & = \text{altitude of cylinder.} \\
\hline
159160 & \\
31832 & \\
\hline
\text{Solidity} = 477\cdot48 & \text{cubic feet.}
\end{array}
$$

4. The circumference of the base of an oblique cylinder
20 feet, and the perpendicular height 18 feet: what is the
lidity?

$$
\begin{array}{r l}
07958 & \\
400 & = 20^2 \\
\hline
31\cdot83200 & \\
18 & \\
\hline
254656 & \\
31832 & \\
\hline
\text{Solidity} = 572\cdot976 & \text{cubic feet} = 573 \text{ cu} \\
& \text{feet, nearly.}
\end{array}
$$

5. Determine the solid content of a right cylinder whose
titude is 15 feet, and the diameter of its base 29 inches.

$$
\begin{array}{r l}
\cdot 7854 & = \text{area of circle whose diameter is 1.} \\
841 & = 29^2 \\
\hline
7854 & \\
31416 & \\
62832 & \\
\hline
660\cdot5214 & = \text{area of cylinder's base, in inches.} \\
180 & \text{inches} = \text{cylinder's altitude.} \\
\hline
528417120 & \\
6605214 & \\
\hline
1728)118893\cdot8520 & \text{cubic inches.} \\
\hline
68\cdot804 & \text{cubic feet.}
\end{array}
$$

Find the content of an oblique cylinder, whose altitude is et; the radius of its base being 10 inches.

$$\begin{array}{rl}
\cdot 7854 & = \text{area of a circle whose diameter is unity.} \\
400 & \\
\hline
314\cdot1600 & \\
114 & \text{inches} = \text{cylinder's perp. altitude.} \\
\hline
12566400 & \\
31416 & \\
31416 & \\
\hline
1728)35814\cdot2400 & \text{cubic inches.} \\
\hline
20\cdot725 & \text{cubic feet.}
\end{array}$$

The circumference of the base of an oblique cylinder is et, and the perpendicular altitude 19·318 feet: what is the ty?

$$\begin{array}{rl}
\cdot07958 & \\
400 & \\
\hline
31\cdot83200 & = \text{area of cylinder's base.} \\
19\cdot318 & = \text{cylinder's altitude.} \\
\hline
254656 & \\
31832 & \\
95496 & \\
286488 & \\
31832 & \\
\hline
\text{Solidity} = 614\cdot930576 & \text{cubic feet.}
\end{array}$$

PROBLEM VI.

To find the convex surface of a right cone.

2. *The diameter of a right cone is 4·5 feet, and the ıeight 20 feet;* required the convex surface.

$$\begin{array}{rl} 3{\cdot}1416 & = \text{circumf. of a circle to di} \\ 4{\cdot}5 & = \text{diam. of cone's base.} \\ \hline 157080 & \\ 125664 & \\ \hline 14{\cdot}13720 & = \text{circumference of cone's} \\ 10 & = \tfrac{1}{2} \text{ slant side.} \\ \hline \end{array}$$

Convex surface = 141·372 sq. feet.

3. The circumference of the base of a right cone is 10· its slant height is 18·9; what is the convex surface?

$$\begin{array}{rl} 10{\cdot}7 & = \text{circumf. of cone's base} \\ 9{\cdot}45 & = \tfrac{1}{2} \text{ slant height.} \\ \hline 535 & \\ 428 & \\ 963 & \\ \hline \end{array}$$

Convex surface = 101·115 as required.

4. Determine the number of square feet contained i outward surface of a square pyramid, each side of its base $3\frac{1}{2}$ feet, and a perpendicular from the vertex upon one c sides of the base being $8\frac{3}{4}$ feet.

$3\frac{1}{2}$ feet = 3·5 feet.

$$\begin{array}{rl} 8{\cdot}75 & = \text{perpendicular.} \\ \hline 4375 & \\ 2625 & \\ \hline 2)30{\cdot}625 & \\ \hline 15{\cdot}3125 & = \text{area of one face of pyramid} \\ 4 & \\ \hline \end{array}$$

Surface = 61·2500 sq. feet.

. triangular pyramid has each side of its base equal to
t, and its slant height 7·25 feet: determine the number
ıre feet of canvass that would be sufficient to cover its
d surface.

```
        5·5
       17·25
      ------
        8625
       8625
      ------
    2)94·875
      ------
     47·4375 = area of one triangular face.
           3
    --------
    142·3125 = number of sq. feet required.
    --------
```

PROBLEM VII.

d the surface of the frustum of a right cone or pyramid.

2. What is the convex surface of the frustum of a right
ıe circumference of the greater end being 29 feet, that
esser end 14 feet, and the length of the slant side 20

```
     29  = circumf. of greater end.
     14  = circumf. of lesser end.
     --
     43
      20 = slant side of the frustum.
    ----
   2)860
    ----
     430 = convex surface required.
    ----
```

ind the convex surface of the frustum of a right cone;
neters of the ends being 9 and 4 feet, and the length of
side 18 feet.

$$\begin{array}{rl}
3{\cdot}1416 & = \text{circumf. of circle to diameter 1.} \\
9 & \\
\hline
28{\cdot}2744 & = \text{circumf. of greater end.} \\
12{\cdot}5664 & = \text{circumf. of lesser end.} \\
\hline
40{\cdot}8408 & \\
18 & = \text{slant side.} \\
\hline
3267264 & \\
408408 & \\
\hline
2)735{\cdot}1344 & \\
\hline
\text{Surface} = 367{\cdot}5672 & \text{sq. feet.}
\end{array}$$

4. If a segment of 7 feet slant height be cut off the top o a cone whose slant height is 30 feet, and circumference of it base 10 feet; find the surface of the frustum.

By similar triangles.

Diameter of base : diameter of upper section : : 30 : 7
or 10 feet : circumf. of upper section : : 30 : 7
∴ circumference of upper section $= \frac{7}{3} = 2{\cdot}3333$ feet.

$$\begin{array}{rl}
2{\cdot}3333 & \\
10{\cdot} & \\
\hline
12{\cdot}3333 & = \text{sum of perimeters.} \\
23 & = \text{slant height of frustum.} \\
\hline
369999 & \\
246666 & \\
\hline
2)283{\cdot}6659 & \\
\hline
\text{Surface} = 141{\cdot}8329 & \text{sq. feet.}
\end{array}$$

5. The slant height of a cone is 20 feet, and the diamete its base 9 feet; what is the convex surface of a frustum cut by a plane parallel to the cone's base, and distant 4 feet the vertex of the cone?

Radius of cone's base = 4·5 feet.

$\therefore \{20^2 - (4\cdot5)^2\}^{\frac{1}{2}} = 19\cdot487$ = perp. altitude of the cone.

Hence by similar triangles,

19·487 : circumf. of cone's base : : 4 : circumf. of upper sec.

or 19·487 : 28·2744 : : 4 : circumf. of upper section.

$\therefore$ the perimeter of upper end $= \frac{113\cdot0976}{19\cdot487} = 5\cdot803$

 28·2744

Sum of perimeters = 34·0774

Again, by similar triangles,

19·487 : 20 : : 4 : slant height of the little cone whose base is the upper section. This slant height $= \frac{80}{19\cdot487} = 4\cdot105$

$\therefore$ 20—4·105 = 15·895 = slant height of the frustum.

34·0774 = sum of perimeters.
15·895 = slant height of frustum.

```
     1703870
    3066966
   2726192
  1703870
  340774
  ----------
2)541·6602730
  ----------
```

Convex surface= 270·8301365 sq. feet.

6. Determine the surface of the frustum of a pentagonal pyramid, one side of the base being 3 feet, and a side of the upper end $1\frac{3}{4}$ feet; the slant height being 5 feet.

15 feet = perimeter of base.
8·75 feet = perimeter of upper end.

23·75 = sum of perimeters.
5 = slant height of frustum.

2)118·75

Surface required = 59·375 sq. feet.

PROBLEM VIII.

To find the solidity of a cone or pyramid.

Ex. 3. Find the content of a pyramid whose base is a regular duodecagon, a side of which is 3 feet, and the altitude of the pyramid 12 feet.

$$\begin{array}{rl} 11{\cdot}196152 & = \text{area of a duodecagon whose} \\ 9 & \quad\text{side is unity.} \\ \hline 100{\cdot}765368 & \\ 12 & = \text{alt. of pyramid.} \\ \hline 3)1209{\cdot}184416 & \\ \hline \text{Solidity} = \quad 403{\cdot}061472 & \text{cubic feet.} \end{array}$$

4. Required the solidity of a triangular pyramid whose height is 30, and each side of the base 4.

$$\begin{array}{rl} {\cdot}433012 & = \text{area of an equilat.} \triangle \text{ whose} \\ 16 & \quad\text{side is unity.} \\ \hline 2598072 & \\ 433012 & \\ \hline 6{\cdot}928192 & \\ 30 & = \text{altitude.} \\ \hline 3)207{\cdot}845760 & \\ \hline 69{\cdot}281920 & = \text{content required.} \end{array}$$

5. Required the solidity of a square pyramid, each side of whose base is 20, and the perpendicular height 60.

$$\begin{array}{rl} 400 & = \text{area of the base.} \\ 60 & \\ \hline 3)24000 & \\ \hline \text{Solidity} = \quad 8000 & \end{array}$$

What is the solidity of a cone the diameter of whose base inches, and its altitude 15 feet?

$$
\begin{array}{rl}
\cdot 7854 & = \text{area of a circle to diam. 1.} \\
324 & = (18)^2 \\
\hline
31416 & \\
15708 & \\
23562 & \\
\hline
254\cdot 4696 & = \text{area of cone's base.} \\
180 & = \text{altitude, in inches.} \\
\hline
203575680 & \\
2544696 & \\
\hline
3)45804\cdot 5280 & \\
\hline
1728)15268\cdot 176 & \text{cubic inches.} \\
\hline
8\cdot 83575 & \text{cubic feet.}
\end{array}
$$

If the circumference of the base of a cone be 40 feet, and height 50 feet; what is the solidity?

$$
\begin{array}{rl}
\cdot 07958 & \\
1600 & = (40)^2 \\
\hline
4774800 & \\
7958 & \\
\hline
127\cdot 32800 & = \text{area of cone's base.} \\
50 & \\
\hline
3)6366\cdot 40000 & \\
\hline
\text{Solidity} = 2122\cdot 133, & \text{\&c., cubic feet.}
\end{array}
$$

What is the content of a pentagonal pyramid, its height 15 feet, and each side of its base 3 feet?

$$
\begin{array}{rl}
1\cdot720477 & = \text{area of a pentagon whose} \\
9 & = 3^2 \qquad \text{[side is unity.} \\
\hline
15\cdot484293 & = \text{area of pyramid's base.} \\
15 & = \text{altitude.} \\
\hline
77421465 & \\
15484293 & \\
\hline
3)232\cdot264395 & \\
\hline
\text{Solidity} = 77\cdot421465 & \text{cubic feet.}
\end{array}
$$

PROBLEM IX.

To find the solidity of the frustum of a cone whose end diameters and altitude are given.

Ex. 2. What is the solidity of the frustum of a cone, the diameter of the greater end being 6 feet, of the lesser end 2 feet, and the altitude 12 feet?

$$6^3-2^3 = 216-8 = 208$$

$$\text{and } 6-2 = 4$$

$$\therefore \frac{6^3-2^3}{6-2} = \frac{208}{4} = 52$$

$$
\begin{array}{rl}
52 & \\
\cdot7854 & \\
\hline
15708 & \\
39270 & \\
\hline
4\cdot08408 & \\
4 & = \tfrac{1}{3} \text{ altitude.} \\
\hline
\text{Solidity} = 163\cdot3632 & \text{cubic feet.}
\end{array}
$$

3. Determine the solidity of the frustum of a cone, whose end diameters are 8 and 6 feet respectively; the altitude of the frustum being 9 inches.

$8^3—6^3 = 512—216 = 296$
and $8—6 = 2$

$\therefore \frac{8^3—6^3}{8—6} = \frac{296}{2} =$ 148
·7854

62832
31416
7854

116·2392
·25 = $\frac{1}{4}$ of a foot = $\frac{1}{3}$ of 9 inches.

5811960
2324784

Solidity = 29·059800 cubic feet.

If the end diameters of a conical frustum be 9 and 7 respectively, and the altitude 4 inches; what is the solid ιt of the frustum?

$9^3—7^3 = 729—343 = 386$
$9—7 = 2$

$\therefore \frac{9^3—7^3}{9—7} = \frac{386}{2} =$ 193
·7854

23562
70686
7854

151·5822

w $\frac{1}{3}$ the altitude = $\frac{1}{3}$ of 4 = $\frac{4}{3}$; hence 151·5822 × $\frac{4}{3}$ 2·1096 cubic inches = the solidity required.

PROBLEM X.

d the solidity of the frustum of a pyramid; supposing the 's of the solid to be regular polygons, and the altitude of frustum to be known.

Ex. 2. What is the solidity of the frustum of a square pyramid; a side of the greater end being 4 feet, that of the lesser end 2 feet, and the altitude 9 feet?

$$4^2 = \text{area of the greater end.}$$
$$2^2 = \text{area of the lesser end.}$$
$$4 \times 2 = \sqrt{4^2 \times 2^2}$$
$$\text{Sum} = 28$$
$$3 = \tfrac{1}{3} \text{ of the altitude.}$$
$$\text{Solidity} = 84 \text{ cubic feet.}$$

3. What is the solidity of the frustum of a square pyramid one side of the greater end being 18 inches, that of the lesser end 15 inches, and the height 30 inches?

$$18^2 = 324 = \text{area of larger end.}$$
$$15^2 = 225 = \text{area of lesser end.}$$
$$18 \times 15 = 270 = \sqrt{18^2 \times 15^2}$$
$$\text{Sum} = 819$$
$$10 = \tfrac{1}{3} \text{ of the altitude.}$$
$$\text{Solidity} = 8190 \text{ cubic inches.}$$

4. What is the solidity of the frustum of an hexagonal pyramid, the side of whose greater end is 8 feet, that of the less end 2 feet, and the height 9 feet?

$$2{\cdot}598076 = \text{area of a hexagon whose side is unity.}$$
$$64 = 8^2$$
$$10392304$$
$$15588456$$
$$166{\cdot}276864 = \text{area of the greater end.}$$
$$2{\cdot}598076$$
$$4 = 2^2$$
$$10{\cdot}392304 = \text{area of the lesser end}$$

Also 166·276864 × 10·392304 = 1727·999718854656
the square root of which = 41·569216

therefore 166·276864
10·392304
41·569216

218·238384
3 = $\frac{1}{3}$ the altitude.

Solidity = 654·715152 cubic feet.

5. Determine the solid content of the frustum of a pyramid, whose base is a regular decagon, a side of which is 4 feet, and side of the upper end 2 feet; the altitude being 15 feet.

7·694208 = area of a decagon whose side is 1.
16 = 4^2

46165248
7694208

123·107328 = area of greater end.
7·694208
4 = 2^2

30·776832 = area of lesser end.

Also 123·107328 × 30·776832 = 3788·853551824896
the square root of which = 61·553664

Hence 123·107328
30·776832
61·553664

215·437824
5 = $\frac{1}{3}$ the altitude.

Solidity = 1077·189120 cubic feet.

PROBLEM XI.

To find the solidity of a wedge.

Ex. 2. The length and breadth of the base of a wedge are 15 and 8 inches respectively, and the length of the edge is 45 inches: *determine its solidity, supposing the altitude of the wedge to be 3 feet.*

15 = length of the b
2

30
45 = length of the e

75
36 = height of the v

450
225

2700
8 = breadth of th

6)21600

Solidity = 3600 cubic inches.

: length and breadth of the base of a wedge
iches respectively, also the length of the wed
determine the solidity, supposing the altitud
be 18 inches.

45 = length of the
2

90
50 = length of the

140
18 = height of the

1120
140

2520
16 = breadth of t

15120
2520

6)40320

Solidity = 6720 cubic inches.

PROBLEM XII.

To find the solidity of a prismoid.

Ex. 2. What is the solid content of a prismoid, whose greater end measures 12 inches by 8; the lesser end 8 inches by 6; and the perpendicular altitude 15 feet?

$$12 \times 8 = 96 = \text{area of greater end.}$$
$$8 \times 6 = 48 = \text{area of lesser end.}$$
$$4 \times 10 \times 7 = 280 = 4 \text{ times area of middle section.}$$

$$424$$
$$30$$

$$1728)12720 \text{ cubic inches.}$$

$$\text{Solidity} = 7{\cdot}3611$$

3. Determine the solid contents of a prismoid whose base measures 9 feet by $3\frac{1}{2}$; and whose upper end measures 12 feet by $4\frac{1}{2}$; the altitude being 9 feet.

$$9 \times 3\tfrac{1}{2} = 31{\cdot}5 = \text{area of smaller end.}$$
$$12 \times 4\tfrac{1}{2} = 54{\cdot} = \text{area of larger end.}$$
$$4 \times \tfrac{21}{2} \times \tfrac{8}{2} = 168 = 4 \text{ times area of middle section.}$$

$$253{\cdot}5$$
$$1{\cdot}5 = \tfrac{1}{6} \text{ of the altitude, in feet.}$$

$$12675$$
$$2535$$

$$380{\cdot}25 = \text{contents.}$$

4. Find the number of imperial gallons contained in a brewer's guile tun in the form of a prismoid, whose greater end measures 12 feet by 8, and lesser end measures 9 feet by 5; the altitude being 9 feet.

$12 \times 8 = 96$ = area of greater end.
$9 \times 6 = 54$ = area of lesser end.
$4 \times \frac{21}{2} \times \frac{14}{2} = 294$ = 4 times the area of the middle section.

$$
\begin{array}{r}
444 \\
1{\cdot}5 \\
\hline
2220 \\
444 \\
\hline
666{\cdot}0
\end{array}
$$

$1{\cdot}5 = \frac{1}{6}$ of the altitude.

Solidity = $666{\cdot}0$ cubic feet.

Now 1 cubic foot = 1728 cubic inches = $\frac{1728}{277{\cdot}274}$ imperial gallons = $6{\cdot}232$ imperial gallons; hence $666 \times 6{\cdot}232 = 4150{\cdot}512$ imperial gallons = the content required.

PROBLEM XIII.

To find the convex surface of a sphere.

Ex. 2. What is the convex superficies of a sphere whose diameter is $2\frac{1}{2}$ feet?

$$
\begin{array}{r}
3{\cdot}1416 \\
2{\cdot}5 \\
\hline
157080 \\
62832 \\
\hline
7{\cdot}85400 \\
2{\cdot}5 \\
\hline
39270 \\
15708 \\
\hline
19{\cdot}6350
\end{array}
$$

$3{\cdot}1416$ = circumf. of a circle to diameter 1.

$7{\cdot}85400$ = circumf. of a circle to diam. $2\frac{1}{2}$.

$19{\cdot}6350$ = surface of the sphere.

What is the convex superficies of a sphere whose circumference is $5{\cdot}19$ feet?

```
3·1416)5·1900(1·652
        31416
        204840
        188496
        ------
         163440
         157080
         ------
           63600
           62832
           -----
             768
```

1·652 = diameter of the sphere.
∴ 5·19 = circumference.

```
 14868
 1652
8260
8·57388
```

8·57388 sq. feet = the surface of the sphere.

. If the diameter of the earth be $7957\frac{3}{4}$ miles, what is the
a of the whole surface, supposing the earth to be a perfect
ere?

```
Earth's diameter = 7957·75
                      3·1416
                   ---------
                     4774650
                     795775
                   3183100
                   795775
                 2387325
                 ------------
                 25000·067400 = earth's circumf.
                     7957·75
                 ------------
                 1250003370
                1750004718
               1750004718
              1250003370
             2250006066
            1750004718
Surface = 198944286·352350 square miles.
```

5. The diameter of a sphere is 21 inches; what is the vex superficies of that segment of it whose height is 9 inc

$$
\begin{array}{rl}
3{\cdot}1416 & = \text{circumf. of circle to diameter 1.} \\
21 & \\
\hline
31416 & \\
62832 & \\
\hline
65{\cdot}9736 & = \text{circumference of the sphere.} \\
9 & \\
\hline
\text{Surface} = 593{\cdot}7624 & \text{square inches.}
\end{array}
$$

6. What is the convex surface of a spherical zone v breadth is 8 inches, and the diameter of the sphere 30 in

$$
\begin{array}{rl}
3{\cdot}1416 & \\
30 & \\
\hline
94{\cdot}2480 & = \text{circumf. of the sphere.} \\
8 & = \text{breadth of the zone.} \\
\hline
\text{Surface} = 753{\cdot}9840 & \text{square inches.}
\end{array}
$$

PROBLEM XIV.

To find the solidity of a sphere.

Ex. 2. What is the solidity of a sphere whose diame $2\frac{2}{3}$ feet?

$$2\tfrac{2}{3} = \tfrac{8}{3}$$

$$\text{and } (\tfrac{8}{3})^3 = \tfrac{512}{27} = 18{\cdot}962 \text{ feet} = (\text{diamete}$$

$$
\begin{array}{r}
18{\cdot}962 \\
{\cdot}5236 \\
\hline
113772 \\
56886 \\
37924 \\
94810 \\
\hline
\end{array}
$$

Solidity $= 9{\cdot}9285032$ cubic inches.

. What is the solidity of the earth, supposing it to be per·
ly spherical, and its diameter equal to 7957¾ miles?

$$
\begin{array}{lr}
\text{Diameter} = & 7957{\cdot}75 \\
 & 7957{\cdot}75 \\
\hline
 & 3978875 \\
 & 5570425 \\
 & 5570425 \\
 & 3978875 \\
 & 7161975 \\
 & 5570425 \\
\hline
 & 63325785{\cdot}0625 \\
 & 7957.75 \\
\hline
 & 3166289253125 \\
 & 4432804954375 \\
 & 4432804954375 \\
 & 3166289253125 \\
 & 5699320655625 \\
 & 4432804954375 \\
\hline
 & 503930766081{\cdot}109375 \\
 & {\cdot}5236 \\
\hline
 & 3023584596486656250 \\
 & 1511792298243328125 \\
 & 1007861532162218750 \\
 & 2519653830405546875 \\
\hline
 & 263858149120{\cdot}0688687500 \text{ cubic miles.} \\
\hline
\end{array}
$$

|. Determine the solid content of a sphere whose circumfe
:e *is 15 feet.*

$$
\begin{array}{l}
3{\cdot}1416)15{\cdot}\quad(4{\cdot}7746 = \text{diameter.} \\
\quad 125664 \\
\hline
\quad 243360 \\
\quad 219912 \\
\hline
\quad\ 234480 \\
\quad\ 219912 \\
\hline
\quad\ \ 145680 \\
\quad\ \ 125664 \\
\hline
\quad\ \ \ 20016
\end{array}
$$

Now $(4{\cdot}7746)^3 = 108{\cdot}845 = (\text{diameter})^3$ nearly

$$
\begin{array}{r}
108{\cdot}845 \\
{\cdot}5236 \\
\hline
653070 \\
326535 \\
217690 \\
544225 \\
\hline
56{\cdot}9912420
\end{array}
$$

Solidity $= 56{\cdot}9912420$ cubic feet.

5. Find the solidity of a sphere whose radius is 18

Diameter $= 36$

$(\text{diameter})^3 = 46656$

$$
\begin{array}{r}
46656 \\
{\cdot}5236 \\
\hline
279936 \\
139968 \\
93312 \\
233280 \\
\hline
24429{\cdot}0816
\end{array}
$$

Solidity $= 24429{\cdot}0816$ cubic feet.

)etermine the content of a sphere whose circumference ches.

```
31416(2·0000000(·63661
    188496
    ------
     115040
      94248
     ------
      207920
      188496
      ------
       194240
       188496
       ------
         5744
         ----
```

Diameter of the sphere = ·63661 inch.

```
Now (·63661)³ =  ·25800039 nearly.
                     ·5236
                 ----------
                 154800234
                 77400117
                51600078
               129000195
               ------------
Contents = ·135089004204 cubic inch.
```

PROBLEM XV.

To find the solidity of the segment of a sphere.

2. Determine the solidity of the segment of a sphere *base diameter is* 25 inches, and its altitude 8 inches.

Radius of the segment's base = 12·5

$$
\begin{array}{r l}
12\cdot5 & \\
\hline
625 & \\
1500 & \\
\hline
156\cdot25 & \\
3 & \\
\hline
468\cdot75 & \\
64 & = \text{(height)}^2 \\
\hline
532\cdot75 & \\
8 & \\
\hline
4262\cdot00 & \\
\cdot5236 & \\
\hline
25572 & \\
12786 & \\
8524 & \\
21310 & \\
\hline
1728)2231\cdot5832 & \text{cubic inches.} \\
\hline
1\cdot291 & \text{\&c., cubic feet.}
\end{array}
$$

3. What is the content of a spherical segment whose h is 6 inches and the radius of its base 9 inches?

Radius of segment's base = 9 inches.

$$
\begin{array}{r}
9 \\
\hline
81 \\
3 \\
\hline
243 \\
36 \\
\hline
279 \\
6 \\
\hline
1674 \\
\cdot5236 \\
\hline
10044 \\
5022 \\
3348 \\
8370 \\
\end{array}
$$

Solidity = 876·5064 cubic inches.

Find the solidity of a spherical segment, the diameter of
e base is 18, and its altitude 5.

dius of segment's base = 9

$$
\begin{array}{r}
9 \\
\hline
81 \\
3 \\
\hline
243 \\
25 \\
\hline
268 \\
5 \\
\hline
1340 \\
\cdot5236 \\
\hline
8040 \\
4020 \\
2680 \\
6700 \\
\hline
701\cdot6240
\end{array}
$$

25 = (height)^2

Solidity = 701·6240 cubic inches.

What is the solidity of a spherical segment, the radius o
se base is 2 inches, and altitude $1\frac{3}{4}$ inches?

Radius of base = 2

$$
\begin{array}{r}
2 \\
\hline
4 \\
3 \\
\hline
12 \\
3\cdot0625 \\
\hline
15\cdot0625 \\
1\cdot75 \\
\hline
10553125 \\
1504375 \\
150625 \\
\hline
26\cdot359375 \\
\cdot5236 \\
\hline
158156250 \\
79078125 \\
52718750 \\
131796875 \\
\hline
13\cdot8017687500
\end{array}
$$

3·0625 = $(1\cdot75)^2$ = (height)^2

Content = 13·8017687500 cubic inches.

PROBLEM XVI.

To find the content of a sector of a sphere whose radius i

Ex. 2. Having given the radius of a sphere = inches, and the altitude of the segment 15 inches, d the content of the sector.

$$\begin{array}{r l} 12{\cdot}25 & = (3{\cdot}5)^2 = (\text{rad.})^2 \\ 1{\cdot}25 & = \text{altitude of the segr} \\ \hline 6125 & \\ 2450 & \\ 1225 & \\ \hline 15{\cdot}3125 & \\ 2{\cdot}0944 & \\ \hline 612500 & \\ 612500 & \\ 1378125 & \\ 3062500 & \\ \hline \end{array}$$

Solidity = $32{\cdot}07050000$ cubic feet.

3. If the diameter of a sphere be 4 feet 3 inches, and titude of the segment 1 foot 6 inches, find the solidity sector.

$$\begin{array}{r l} 4{\cdot}515625 & = (2{\cdot}125)^2 = (\text{radiu} \\ 1{\cdot}5 & = \text{altitude of the seg} \\ \hline 22578125 & \\ 4515625 & \\ \hline 6{\cdot}7734375 & \\ 2{\cdot}0944 & \\ \hline 270937500 & \\ 270937500 & \\ 609609375 & \\ 1354687500 & \\ \hline \end{array}$$

$14{\cdot}18628750000$ = solidity in cubic

PROBLEM XVII.

y given the end diameters of a frustum or zone of a sphere and the altitude of the frustum, to find its solidity.

2. Find the solid content of a zone, whose greater dia- is 22 inches, lesser diameter 16 inches, and the distance two ends 9 inches.

$$\begin{array}{rl} \text{Here } 121 & = (\text{greater radius})^2 \\ 64 & = (\text{lesser radius})^2 \\ 27 & = \frac{1}{3}\,(\text{altitude})^2 \\ \hline 212 & \\ 9 & = \text{the altitude.} \\ \hline 1908 & \\ 1{\cdot}5708 & \\ \hline 15264 & \\ 133560 & \\ 9540 & \\ 1908 & \\ \hline \text{Solidity} = 2997{\cdot}0864 & \text{cubic inches.} \end{array}$$

Determine the solidity of the middle zone of a sphere end diameters are 3 feet each, and the breadth of the , feet.

1·5 feet = radius of each end.

$$\begin{array}{rl}
(1\cdot5)^2 & = 2\cdot25 \\
 & 2\cdot25 \\
\frac{1}{3}\ (\text{altitude})^2 & = 5\cdot3333 \\
\hline
 & 9\cdot8\dot{3}33 \\
 & 4 = \text{altitude.} \\
\hline
 & 39\cdot3332 \\
 & 1\cdot5708 \\
\hline
 & 3146656 \\
 & 2753324 \\
 & 1966660 \\
 & 393332 \\
\hline
\text{Solidity} & = 61\cdot78459056 \text{ cubic feet.}
\end{array}$$

4. Find the number of cubic feet contained in a sph
zone whose top and bottom diameters are 3 feet, and
respectively, and their distance 4 feet.

$$\begin{array}{rll}
(1\cdot5)^2 = & 2\cdot25 & = (\text{greater radius})^2 \\
 & 1\cdot & = (\text{lesser radius})^2 \\
 & 5\cdot3333 & = \frac{1}{3}\ (\text{altitude})^2 \\
\hline
 & 8\cdot5833 & \\
 & 4 & = \text{altitude.} \\
\hline
 & 34\cdot3332 & \\
 & 1\cdot5708 & \\
\hline
 & 2746656 & \\
 & 2403324 & \\
 & 1716660 & \\
 & 343332 & \\
\hline
= & 53\cdot93059056 & \text{cubic feet.}
\end{array}$$

. The end diameters of a zone are 12 and 10 feet; also the :ude of the zone is 6 feet: determine its solidity.

$$6^2 = 36 = (\text{greater rad.})^2$$
$$5^2 = 25 = (\text{lesser rad.})^2$$
$$\tfrac{1}{3} \text{ of } 6^2 = 12 = \tfrac{1}{3}\,(\text{altitude})^2$$

$$\begin{array}{r}
73 \\
6 = \text{altitude.} \\
\hline
438 \\
1{\cdot}5708 \\
\hline
125664 \\
47124 \\
62832 \\
\hline
\end{array}$$

Solidity $= 688{\cdot}0104$ cubic feet.

PROBLEM XVIII.

find the solidity of a spheroid, whose major and minor axes are known.

:x. 2. What is the solidity of a prolate spheroid, whose fix- ıxis is 90, and its revolving or minor axis 70?

$$\begin{array}{r}
70 = \text{revolving or minor axis.} \\
70 \\
\hline
4900 = (\text{minor axis})^2 \\
90 = \text{major axis.} \\
\hline
441000 \\
{\cdot}5236 \\
\hline
5236 \\
20944 \\
20944 \\
\hline
\end{array}$$

Solidity $= 230907{\cdot}6000$

3. Determine the content of an oblate spheroid whos
or minor axis is 70, and whose revolving or major axis

```
                 90 = revolving axe.
                 90
               ----
               8100
                 70  = fixed axe.
             ------
             567000
             ·5236
           --------
           36652000
          31416
         26180
        -----------
Solidity = 296881·2000
        -----------
```

4. The major and minor axes are 304 and 503; dete
the content of the oblate spheroid, and how much this
tent differs from that of a sphere on the major axis.

```
                304 = revolving axe.
                304
               ----
               1216
              912
             ------
              92416 = (revolv. axe)²
                303
             ------
             277248
           277248
           --------
           28002048
              ·5236
         ----------
          168012288
          84006144
         56004096
       140010240
       ------------
       14661872·3328 = content of oblate s
```

$$
\begin{array}{r}
\text{Now } (304)^3 = 28094464 \\
\cdot 5236 \\
\hline
168566784 \\
84283392 \\
56188928 \\
140472320 \\
\hline
14710261 \cdot 3504 = \text{cont. of sphere on maj. axis.} \\
14661872 \cdot 3328 = \text{cont. of obl. spheroid.} \\
\hline
48389 \cdot 0176 = \text{diff. of their solidities.}
\end{array}
$$

PROBLEM XIX.

To find the solidity of the segment of a spheroid.

CASE I.—*When the base is perpendicular to the fixed axis.*

Ex. 2. The axes of a prolate spheroid are 40 and 30 inches respectively; what is the solidity of that segment whose height is 6 inches, and its base perpendicular to the fixed, or, in this case, the major axis?

$\frac{30^2}{40^2} = \frac{9}{16} = \cdot 5625$

$$
\begin{array}{rr}
\text{Three times the fixed axis} = & 120 \\
\text{Twice the height of the seg.} = & 12 \\
\hline
\text{Difference} = & 108 \\
& \cdot 5625 \\
\hline
& 45000 \\
& 5625 \\
\hline
& 60 \cdot 7500 \\
(\text{height})^2 = & 36 \\
\hline
& 36450 \\
& 18225 \\
\hline
& 2187 \cdot 00 \\
& \cdot 5236 \\
\hline
& 13122 \\
& 6561 \\
& 4374 \\
& 10935 \\
\hline
\textit{Solidity} = & 1145 \cdot 1132 \text{ cubic inches.}
\end{array}
$$

3. The major and minor axes of an oblate spheroid a
and 60 respectively; determine the solid content of tha
tion of it whose altitude is 8 inches, and its base perpen
to the fixed, or, in this case, minor axis.

$$\frac{100^2}{60^2} = \frac{100}{36} = \frac{25}{9} = 2{\cdot}7777$$

$$
\begin{array}{rr}
\text{Three times the fixed axis} = & 180 \\
\text{Twice the height} = & 16 \\
\hline
\text{Difference} = & 164 \\
 & 2{\cdot}7777 \\
\hline
 & 111108 \\
 & 166662 \\
 & 27777 \\
\hline
 & 455{\cdot}5428 \\
(\text{altitude})^2 = & 64 \\
\hline
 & 18221712 \\
 & 27332568 \\
\hline
 & 29154{\cdot}7392 \\
 & {\cdot}5236 \\
\hline
 & 1749284352 \\
 & 874642176 \\
 & 583094784 \\
 & 1457736960 \\
\hline
\text{Solidity} = & 15265{\cdot}42144512 \text{ cubic inches.} \\
1728) & \overline{8{\cdot}8341, \text{\&c. cubic feet.}}
\end{array}
$$

4. The transverse and conjugate axes of an oblate s
ar feet respectively; find the solidity of that s
is 3; its base being perpendicular to the

$\frac{144}{64} = 2{\cdot}25$

Three times the fixed axis $= 24$ feet.
Twice the height of the seg. $= 6$ feet.

$$
\begin{array}{rr}
\text{Difference} = & 18 \\
 & 2{\cdot}25 \\
\hline
 & 1800 \\
 & 225 \\
\hline
 & 40{\cdot}50 \\
 & 9 \quad = (\text{height})^2 \\
\hline
 & 364{\cdot}5 \\
 & {\cdot}5236 \\
\hline
 & 21870 \\
 & 10935 \\
 & 7290 \\
 & 18225 \\
\hline
\text{Solidity} = & 190{\cdot}85220 \text{ cubic feet.}
\end{array}
$$

CASE II. — *When the base is perpendicular to the revolving axis.*

Ex. 2. Required the solid content of the segment of a prolate spheroid, whose axes are 40 and 30; the altitude of th segment being 6.

$\frac{40}{30} = \frac{4}{3}$

Three times the revolving axes $= 90$
Twice the height of segment $= 12$

Difference $= 78$

$$\begin{array}{r}
\text{And } 78 \times \tfrac{4}{3} = 104 \\
36 = (\text{height})^2 \\
\hline
624 \\
312 \\
\hline
3744 \\
\cdot 5236 \\
\hline
22464 \\
11232 \\
7488 \\
18720 \\
\hline
\text{Solidity} = 1960\cdot 3584 \\
\hline
\end{array}$$

3. Determine the solid content of the segment of spheroid whose axes are 36 and 24; the altitude bei

$$\tfrac{24}{36} = \tfrac{2}{3}$$

$$\begin{array}{lr}
\text{Three times the revolving axis} = & 108 \\
\text{Twice the height of segment} = & 16 \\
\hline
 & 192
\end{array}$$

$$\begin{array}{r}
\text{And } 192 \times \tfrac{2}{3} = 128 \\
64 = (\text{height})^2 \\
\hline
512 \\
768 \\
\hline
8192 \\
\cdot 5236 \\
\hline
49152 \\
24576 \\
16384 \\
40960 \\
\hline
\text{Solidity} = 4289\cdot 3312 \\
\hline
\end{array}$$

. Find the solidity of a segment of a prolate spheroid, ›se axes are 36 and 24; the altitude being 12.

‚ince the minor axis is 24, and the altitude of the segment is the content will evidently be one half of the prolate spheroid.

$\frac{36}{24} = \frac{3}{2}$

ee times the revolving axis = 72
ce height of segment = 24

Difference = 48

And $48 \times \frac{3}{2} =$ 72
144 = $(\text{height})^2$

288
1008

10368
·5236

62208
31104
20736
51840

Solidity = 5428·6848

PROBLEM XX.

ɔ *find the content of the middle frustum of a spheroid, the* '*h, middle diameter, and the diameter of either end being* ı.

ᴀsᴇ I.—*When the ends are circles, whose planes are par- to the revolving axis, and therefore perpendicular to the* ' *axis.*

ᴋ. 2. What is the solidity of the middle frustum of a pro- *spheroid, the* middle diameter being 40, that of either of *wo ends 30,* and the distance of the two ends 50?

$$
\begin{array}{rl}
40 & \\
40 & \\
\hline
1600 & = \text{(middle diam.)}^2 \\
2 & \\
\hline
3200 & = 2\ \text{(midd. diam.)}^2 \\
900 & = \text{(end diam.)}^2 \\
\hline
4100 & \\
50 & = \text{length of the fr} \\
\hline
205000 & \\
\cdot 2618 & \\
\hline
13090000 & \\
5236 & \\
\hline
\text{Solidity} = 53669\cdot 0000 &
\end{array}
$$

3. What is the solidity of the middle frustum of ar spheroid, the middle diameter being 100, that of eithe ends 80, and the distance of the ends 36?

$$
\begin{array}{rl}
100 & \\
100 & \\
\hline
10000 & \\
2 & \\
\hline
20000 & = 2\ \text{(midd. diam.)}^2 \\
6400 & = \text{(end diam.)}^2 \\
\hline
26400 & \\
36 & \\
\hline
158400 & \\
79200 & \\
\hline
950400 & \\
\cdot 2618 & \\
\hline
76032 & \\
9504 & \\
57024 & \\
19008 & \\
\hline
\text{Solidity} = 248814\cdot 72 &
\end{array}
$$

Determine the solidity of a middle spheroidal frustum, e greatest diameter is 24 inches, the diameters of either ieing 16 inches, and the length of the frustum 30 inches; pheroid being oblate.

$$
\begin{array}{rl}
24^2 = 576 & \\
2 & \\
\hline
1152 & = 2\,(\text{midd. diam.})^2 \\
256 & = (\text{end diam.})^2 \\
\hline
1408 & \\
30 & = \text{length of the frustum.} \\
\hline
42240 & \\
\cdot 2618 & \\
\hline
337920 & \\
4224 & \\
25344 & \\
8448 & \\
\hline
1728)11058\cdot 4320 & \text{cubic inches.} \\
\hline
6\cdot 3995 & \text{cubic feet.}
\end{array}
$$

II.—*When the ends are ellipses whose planes are perpendicular to the revolving axis.*

ɛ. 2. The major and minor axes of the middle section of a .te spheroidal frustum are 80 and 60 respectively; those ther end are 60 and 45; determine the solidity of the frusits height being 39·686.

$$
\begin{array}{rl}
160 & = 2 \text{ major axis.} \\
60 & = \text{minor axis.} \\
\hline
9600 & \\
2700 & = \text{product of 60 and 45.} \\
\hline
12300 & \\
39{\cdot}686 & = \text{distance of the ends.} \\
\hline
11905800 & \\
79372 & \\
39686 & \\
\hline
488137{\cdot}800 & \\
{\cdot}2618 & \\
\hline
39051024 & \\
4881378 & \\
29288268 & \\
9762756 & \\
\hline
127794{\cdot}47604 & = \text{solidity.}
\end{array}
$$

3. The major and minor axes of the middle section of a late spheroidal frustum are 90 and 60 respectively; the either end are 60 and 40; determine the solidity of the tum, its height being 30.

$$
\begin{array}{rl}
180 & = 2 \text{ major axis.} \\
60 & = \text{minor axis.} \\
\hline
10800 & \\
2400 & = 60 \times 40 \\
\hline
13200 & \\
30 & = \text{altitude.} \\
\hline
396000 & \\
{\cdot}2618 & \\
\hline
15708000 & \\
23562 & \\
7854 & \\
\hline
\text{Solidity} = 103672{\cdot}8000 &
\end{array}
$$

two axes of the middle section of a prolate spheroidal
re 24 and 18 respectively; those of either end are
find the solidity, its altitude being 10.

$$
\begin{array}{rl}
48 & = 2 \text{ major axis.} \\
18 & = \text{minor axis.} \\
\hline
384 & \\
48 & \\
\hline
864 & \\
48 & = \text{product of } 8 \text{ and } 6. \\
\hline
912 & \\
10 & = \text{dist. between the ends.} \\
\hline
9120 & \\
\cdot 2618 & \\
\hline
52360 & \\
2618 & \\
23562 & \\
\hline
\text{Content} = 2387 \cdot 6160 &
\end{array}
$$

PROBLEM XXI.

To find the solidity of a paraboloid.

What is the solidity of a paraboloid, whose height is
he diameter of its circular base 100?

$$
\begin{array}{rl}
\cdot 7854 & = \text{area of a circle to diam. 1.} \\
10000 & = 100^2 \\
\hline
7854 \cdot 0 & = \text{area of the solid's base.} \\
30 & = \text{altitude.} \\
\hline
2)235620 & \\
\hline
117810 & = \text{content required.}
\end{array}
$$

3. Required the solidity of a paraboloid, whose height is 6
and the diameter of its base 40.

$$
\begin{array}{rl}
\cdot 7854 & = \text{area of a circle to diam.} \\
1600 & = 40^2 \\
\hline
4712400 & \\
7854 & \\
\hline
1256{\cdot}6400 & = \text{area of solid's base.} \\
60 & = \text{altitude of paraboloid.} \\
\hline
2)75398{\cdot}4 & \\
\hline
37699{\cdot}2 & = \text{content required.}
\end{array}
$$

4. Required the solidity of a paraboloid, whose height is
and the diameter of its base 50.

$$
\begin{array}{rl}
\cdot 7854 & = \text{area of circle to diameter} \\
2500 & = 50^2 \\
\hline
3927000 & \\
15708 & \\
\hline
1963{\cdot}5000 & \\
50 & = \text{altitude.} \\
\hline
2)98175 & \\
\hline
49087{\cdot}5 & = \text{content.}
\end{array}
$$

5. Determine the content of a paraboloid whose axis is
inches long, and the radius of its base 8 inches.

$$
\begin{array}{rl}
\cdot 7854 & = \text{area of circle to diameter} \\
256 & = 16^2 = (\text{diameter})^2 \\
\hline
47124 & \\
39270 & \\
15708 & \\
\hline
201{\cdot}0624 & \\
15 & \\
\hline
10053120 & \\
2010624 & \\
\hline
2)3015{\cdot}9360 & \\
\text{Solidity} = 1507{\cdot}968 & \text{cubic inches.}
\end{array}
$$

6. Find the content of a paraboloid the length of whose axi is 3 feet, and the radius of whose base is 2 feet.

$$\begin{array}{r l} \cdot 7854 & \\ 16 & = 4^2 = (\text{diameter})^2 \\ \hline 47124 & \\ 7854 & \\ \hline 12\cdot5664 & \\ 3 & = \text{altitude.} \\ \hline 2)37\cdot6992 & \\ \hline \end{array}$$

Solidity $= 18\cdot8496$ cubic feet.

PROBLEM XXII.

To find the solidity of the frustum of a paraboloid, when it ends are perpendicular to the axis of the solid.

Ex. 2. What is the solidity of the frustum of a paraboloi or parabolic conoid, the diameters of the two ends being 36 an 24; and altitude 14?

$$\begin{array}{r r l} 36^2 = & 1296 & \\ 24^2 = & 576 & \\ \hline \text{Sum} = & 1872 & \\ & 14 & = \text{height of frustum} \\ \hline & 7488 & \\ & 1872 & \\ \hline & 26208 & \\ & \cdot3927 & \\ \hline & 183456 & \\ & 52416 & \\ & 235872 & \\ & 78624 & \\ \hline \end{array}$$

Solidity $= 10291\cdot8816$

3. Determine the solidity of the frustum of a parabolic co the radii of whose ends are 2 and 3 feet respectively, and tude 4 feet.

$$4^2 = 15$$
$$6^2 = 36$$

52 = sum of squares of diams.
4 = altitude.

208
·3927

31416
7854

Solidity = 81·6816 cubic feet.

4. Find the solidity of the frustum of a paraboloid, th of whose ends are 9 and 16 inches respectively, and the a 1 foot 5 inches.

$$18^2 = 324$$
$$32^2 = 1024$$

1348 = sum of the squares of the c
17 inches = altitude.

9436
1348

22916
·3927

160412
45832
206244
68748

8999.1132 cubic inches.

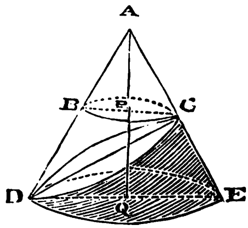
1728)

Solidity = 5·207, &c. cubic feet.

PROBLEM XXIII.

To find the solid content of the elliptic ungula D C E *cut from onical frustum whose end diameters and altitude are given.*

Ex. 2. The base and end diameters of a conical frustum are and 5 inches respectively, also the altitude of the frustum 60 inches; determine the content of the elliptic ungula cut m the frustum.



$45 \times 5 = 225 =$ product of the two diameters.

and $\sqrt{225} = 15$

$5 =$ lesser diameter.

75

$2025 = 45^2 =$ (greater diam.)^2

40)1950

48·75

$2700 =$ greater diam. $\times$ alt.

3412500
9750

131625·00
·2618

1053000
131625
789750
263250

Solidity = 34459·4250 cubic inches.

3. If the two diameters of a conical frustum be 16 a inches respectively, also the altitude of the frustum 21 incl determine the content of the elliptic ungula cut from the f tum, as in Examples 1 and 2, where the ellipse passes thr the edge of the base.

$$\begin{array}{r}
16 \times 4 = 64 = \text{product of the two diameters.} \\
\sqrt{64} = 8 \\
4 = \text{lesser diameter.} \\
\hline
32 \\
256 = 16^2 = \text{(greater diam.} \\
\hline
12)224 \\
\hline
18{\cdot}666, \&c. \\
336 = 16 \times 21 \\
\hline
111996 \\
55998 \\
55998 \\
\hline
6271{\cdot}776 \\
{\cdot}2618 \\
\hline
50174208 \\
6271776 \\
37630656 \\
12543552 \\
\hline
\text{Content} = 1641{\cdot}9509568 \text{ cubic inches.}
\end{array}$$

PROBLEM XXIV.

Having given the length and middle diameter of a circula spindle, to find its solidity.

Ex. 2. If the length of a circular spindle be 50 inches and *middle* diameter 30: what is the solidity?

$$25^2 + 15^2 = 625 + 225 = 850$$

$$\therefore \tfrac{850}{30} = \tfrac{85}{3} = 28.333, \&c. = \text{radius of the circle.}$$

$$15\cdot \qquad = \tfrac{1}{2} \text{ middle diameter.}$$

$$13\cdot333, \&c. = \text{central dist. R O.}$$

Also $\frac{15}{56\cdot666} = \cdot264\,\frac{5}{7}$ nearly $=$ tab. vers. of arc A C.

Now tab. area to $\cdot264 = \cdot165780$
Do. to $\cdot265 = \cdot166663$

```
              ·000883
                    5
            7).004415
              ·000630
              ·165780
```

Tabular area to $\cdot264\frac{5}{7} = \cdot166410$
$3211\cdot03 = (56\cdot666)^2$ nearly.

```
          499230
        166410
       166410
      332820
     499230
  2)534.34750230 = area of segment A C B A
    267·17375115 = ½ generating area.
          13·333 = central distance.
    80152125345
   80152125345
  80152125345
 80152125345
267173751 15
3562·22762408295
5208·33333333333 = ⅓ cube of ½ length.
1646·10570925038
```

And if this be multiplied by 12·5664 the product will be equal to *20685·6226 cubic* inches, the solidity required.

3. Having given the length of a circular spindle equa feet, and its middle diameter 8 inches, to determine its content.

$$4^2 + 12^2 = 160$$
$$\therefore \tfrac{160}{8} = 20 = \text{radius of the circle.}$$
$$20 - 4 = 16 = \text{central distance.}$$
$$\text{Again } \tfrac{4}{40} = \tfrac{1}{10} = \cdot 1$$

And tabular area corr. to ·1 = .040875

$$\begin{array}{rr}
 & .040875 \\
 & 1600 = (\text{diam.})^2 \\
\hline
 & 24525000 \\
 & 40875 \\
\hline
\text{Area of generating segment} = & 65{\cdot}400000 \\
 & 2)\overline{} \\
 & 32{\cdot}7 = \tfrac{1}{2} \text{ generating a} \\
 & 16 = \text{central distanc} \\
\hline
 & 1962 \\
 & 327 \\
\hline
\text{Cent. dist.} \times \tfrac{1}{2} \text{ gener. area} = & 523{\cdot}2 \\
\tfrac{1}{3} \text{ the cube of } 12 = & 576{\cdot} \\
\hline
 & 52{\cdot}8 \\
 & 12{\cdot}5664 \\
\hline
 & 1005312 \\
 & 251328 \\
 & 628320 \\
\hline
\text{Solidity} = & 663{\cdot}50592 \text{ cubic inches.}
\end{array}$$

4. Having given the length of a circular spindle equa inches, and its middle diameter 10 inches; to find th content of the spindle.

$$5^2 + 10^2 = 125$$
$$\therefore \tfrac{125}{10} = 12{\cdot}5 = \text{radius of the circ}$$
$$\text{hence } 12{\cdot}5 - 5 = 7{\cdot}5 = \text{central di}$$
$$\tfrac{5}{25} = \tfrac{1}{5} = {\cdot}2$$

tab. area corr. to ·2 = ·111823
625 = 25^2 = $(\text{diam.})^2$

559115
213646
670938

Generating area = 69.889375
2)

$\frac{1}{2}$ gen. area = 34·944687
Central dist. = 7·5

174723435
244612809

262·0851525
$\frac{1}{3}$ of 10^3 = 333·3333333
71·2481808
12·5664

2849927232
4274890848
4274890848
3562409040
8549781696

Content = 895·33313920512 cubic inches.

PROBLEM XXV.

o find the solidity of the middle frustum of a circular spindle; length of the frustum, its middle diameter, and that of either he ends being given.

x. 2. The middle diameter of the frustum of a circular dle is 32 inches, the diameter at the end 24, and the length nches; determine its solidity.

Middle diameter = 32
End diameter = 24
Difference = 8
2)
4 = $\frac{1}{2}$ difference = vers. of arc NC.

Now diameter $\times$ 4 $= 20^2 + 4^2 = 416$

$\therefore$ diameter of arc ACB $= 104$

Also $52—16 = 36 =$ central distance.

Again $\frac{4}{104} = \frac{1}{26} = \cdot 038\frac{1}{2}$ nearly.

Now tab. area corr. to $\cdot 038\frac{1}{2} = \cdot 009955$

$$\begin{array}{r}
10816 = 104^2 \\
\hline
59730 \\
9955 \\
79640 \\
9955 \\
\hline
\end{array}$$

Area NCP $= 107\cdot 673280$

And $\square$ NE $= 480\cdot$

$\therefore$ area NEePC $= 587.67328$

Again $52^2 = 2704 =$ (radius)^2

$36^2 = 1296 =$ (cent. dist.)^2

$\overline{1408}$

And $\sqrt{1408} = 37\cdot 523$, &c. $= \frac{1}{2}$ length of spindl

1408 $=$ sq. of $\frac{1}{2}$ length of

$133\cdot 3333 = \frac{1}{3}$ sq. of $\frac{1}{2}$ length o

$$\begin{array}{r}
1274\cdot 6667 \\
20 = \tfrac{1}{2}\text{ length of frustu} \\
\hline
25493\cdot 3340 \\
21156\cdot 23808 = \text{general area} \times \text{distance.} \\
\hline
4337\cdot 09592 \\
6\cdot 2832 \\
\hline
867419184 \\
1301128776 \\
3469676736 \\
867419184 \\
2602257552 \\
\end{array}$$

1728)27250·841084544 cubic inches.

15·7701 cubic feet.

The length of the middle frustum of a circular spindle is t 2 inches, the end diameter is 2 feet 1 inch, and the mid-iameter 2 feet 11 inches ; determine its solidity.

Middle diameter = 35 inches.
End diameter = 25 inches.

Difference = 10
2) —
5 = vers. of arc N C.

Also diameter × 5 = $25^2 + 5^2$ = 650
∴ diameter = 130, and rad. = 65
Also 65—17·5 = 47·5 = central distance.

ow $\frac{5}{130} = \frac{1}{26}$ = ·038$\frac{1}{2}$

area corr. to ·038$\frac{1}{2}$ = 009955
16900 = 130^2

8959500
59730
9955

ı of cir. seg. N C P = 168·239500
Area of ▭ N e = 625·

rea of N E e P C = 793·2395 = generating area.

Again 65^2 = 4225 = $(\text{radius})^2$
$(47·5)^2$ = 2256·25 = $(\text{centre dist.})^2$

1968·75

nd $(1968·75)^{\frac{1}{2}}$ = 44·37 = $\frac{1}{2}$ length of spindle.

$$\begin{array}{rrl}
 & 1968{\cdot}75 & = \text{sq. of } \tfrac{1}{2} \text{ length of spindle.} \\
\tfrac{1}{3} \text{ of } 25^2 = & \underline{208{\cdot}3333} & = \tfrac{1}{3} \text{ sq. of half length of frus.} \\
 & 1760{\cdot}4167 & \\
 & \underline{25} & = \tfrac{1}{2} \text{ length of frustum.} \\
 & 88020835 & \\
 & \underline{35208334} & \\
 & 44010{\cdot}4175 & \\
 & \underline{37678{\cdot}87625} & = \text{gen. area} \times \text{cent. dist.} \\
 & 6331{\cdot}54125 & \\
 & \underline{6{\cdot}2832} & \\
 & 1266308250 & \\
 & 1899462375 & \\
 & 5065233000 & \\
 & 1266308250 & \\
 & \underline{3798924750} & \\
1728) & 39782{\cdot}339982000 & \text{cubic inches.} \\
 & 23{\cdot}0222 & \text{cubic feet, nearly.}
\end{array}$$

PROBLEM XXVI.

To find the content of an elliptic spindle.

Ex. 2. The length of an elliptic spindle is 40 inches, the middle diameter 12, and the diameter half way between the end and the middle is 9·49546 inches: determine its solidity.

$$\begin{array}{rl}
3\,\text{CD}^2 = & 432 \\
4\,\text{PQ}^2 = & \underline{360{\cdot}6550424464} \\
\text{Difference} = & 71{\cdot}3449575536 \\
\text{Also } 4\,\text{PQ} = & 37{\cdot}98184 \\
3\,\text{CD} = & \underline{36{\cdot}} \\
\text{Difference} = & 1{\cdot}98184
\end{array}$$

Hence $\dfrac{71{\cdot}3449575536}{1{\cdot}98184} = 36$ nearly, one fourth of which $=$ 9 $=$ the central distance RO.

Now $\text{RC} = \text{RO} + \text{OC} = 9 + 6 = 15 =$ semi-conjugate axis. And from the nature of the ellipse,

$$24 \times 6 : 20^2 :: 15^2 : (\text{semi-transverse axis})^2$$

$$\text{or } 12 : 20 :: 15 : \text{semi-transverse axis.}$$

$$\therefore \text{ semi-transverse} = 25.$$

Also $\frac{6}{30} = \frac{1}{5} = {\cdot}2.$

Also ·111823 = tab. area corr. to ·2
50 = transverse.

5·591150
30 = conjugate.

167·734500 = elliptic space A C B.
3

40)503·203500 = 3 times the area.

12·5800875
12·

·5800875
36 = 4 times the cent. dist.

34805250
17402625

20·8831500
144· = sq. of mid. diameter.

123·11685
13·333 = $\frac{1}{3}$ the length of the spindle.

36935055
36935055
36935055
36935055
12311685

1641·51696105
1·57079

1477365264945
1149061872735
1149061872735
820758480525
164151696105

2578·4784272477295

TE.—It may sometimes be more convenient to multiply by the whole h of the spindle, and lastly to multiply by ·5236, which is $\frac{1}{3}$ of 1·5707 *doing we avoid one decimal factor, and thus obtain a result more* *than by the method adopted in* Ex. 2.

3. If the length of an elliptic spindle be 36 inches, the mi dle diameter 18 inches, and the diameter at 9 inches from t end be 14·1926 inches: find the solidity of the spindle.

$$3\,\text{CD}^2 = 972$$
$$4\,\text{PQ}^2 = 805{\cdot}71957904$$

$$\text{Difference} = 166{\cdot}28042096$$

$$\text{Also } 4\,\text{PQ} = 56{\cdot}7704$$
$$3\,\text{CD} = 54{\cdot}$$

$$\text{Difference} = 2{\cdot}7704$$

Hence $\dfrac{166{\cdot}28042096}{2{\cdot}7704} = 60$ nearly, one fourth of which, 15 = the central distance.

Hence $15 + 9 = 24$ = one of the semi-axes.

And from the nature of the ellipse,

$$39 \times 9 : 18^2 :: 24^2 : (\text{other semi-axis})^2$$

Hence the other semi-axis $= \dfrac{48\sqrt{3}}{\sqrt{13}} = 23$, very nearly.

Consequently the major and minor axes are 48 and 46, and t spindle is evidently described by the revolution of that porti of the ellipse which is at the extremity of the major axis.

Now $\frac{9}{48} = \frac{3}{16} = {\cdot}187\frac{1}{2}$ = tab. vers.

Also the tab. area corr. to $.187\frac{1}{2} = {\cdot}101943$

$$\begin{array}{rl} {\cdot}101943 & \\ 48 & = \text{transverse} \\ \hline 4{\cdot}893264 & \\ 46 & = \text{conj.} \\ \hline \end{array}$$

$$\text{Elliptic area ACB} = 225{\cdot}090144$$
$$3$$

$$\text{3 times the area} = 675{\cdot}270432$$

3 times the area = 675·270432
36)
18·757512
Middle diameter = 18·

·757512
imes the central dist. = 60

45·450720
. of middle diam. = 324·

278·54928
⅓ length of spindle = 12

3342·59136
1·57079

3008332224
2339813952
2339813952
1671295680
334259136

Solidity = 5250·5090823744 cubic inches.

PROBLEM XXVII.

To find the solidity of the middle frustum of an elliptic spindle; length, its diameter at the middle and end being given; also the meter which is half way between the middle and end diameter ng known.

Ex. 2. What is the solid content of the middle frustum of an iptic spindle, its middle diameter being 32 inches, its end diater 24 inches, and the diameter at 10 inches from the end al to 30·15756 *inches;* the whole length of the frustum *ng 40 inches?*

$$3\,CD^2 = 3072$$
$$Nn^2 = 576$$
$$\overline{3648\cdot}$$
$$4 \times (30\cdot15756)^2 = 3637\cdot9136966144$$
$$\overline{10\cdot0863033856}$$

Also $4\,PQ = 120\cdot63024$
$Nn + 3\,CD = 120\cdot$

Difference $= \cdot63024$

Hence $\dfrac{10\cdot0863033856}{\cdot63024} = 16$, nearly.

And $\frac{16}{4} = 4 =$ the central distance.

Now $4 + 16 = 20 =$ semi-conjugate axis.

Also $CR = 16-12 = 4$; and the remainder of the conjugate is therefore 36.

Hence $4 \times 36 : 20^2 :: 20^2 :$ (semi-transverse axis)^2
or $12 : 20 :: 20 :$ semi-transverse.

$\therefore$ semi-transverse $= \frac{400}{12} = 33\cdot33$, &c.
and the transverse $= 66\cdot66$, &c. $= 66\frac{2}{3}$.

Again $\frac{4}{40} = \cdot1$

And $\cdot040875 =$ tabular area corr. to it.
$66\frac{2}{3} =$ transverse axis.

$2\cdot725$
$40 =$ conjugate.

$109\cdot000 =$ elliptic area of which N C m is the arc.
3

$327 =$ three times the elliptic area.

40)327 = three times the elliptic area.

8·175
8· = diff. of middle and end diam.

·175
32 = 8 times the central distance.

350
525

5·600
2624 $= \text{N}\, n^2 + 2\, \text{C}\, \text{D}^2$

2618·4
40 = length of frustum.

104736·0
·2618 = common multiplier.

837888
104736
628416
209472

27419·8848 cubic inches, the solidity required.

---

t.—$\{\Delta^2 + \delta^2 + 4m^2\}$ ·1309L is a very useful formula for the middle
i of a solid formed by the revolution of a conic section about its axis.
representing the greatest and least diameter, m the middle diameter,
he length of the frustum.

ost of the examples of spindles, their frustums, &c., it happens that
tral distance is found to be not a finite number; but we must always
e nearest integer to be the central distance : for when we consider
dom it can happen that the quantity δ is an integer, even though the
an ellipse are integers, it is evident that the fraction $\frac{3\,\text{D}^2 + \Delta^2 - 4\,\delta^2}{4\,\delta - 3\,\text{D} - \Delta}$
almost all cases, be of an indeterminate value.—For all practical
s, the short formula just given, and which may be found in the Ap-
to the Mensuration, Problem XXIX, in a note, is recommended as
tly exact.

PROBLEM XXVIII.

To find the content of a parabolic spindle.

Ex. 2. The length of a parabolic spindle is 15 i
the middle diameter is one foot: determine the con
spindle.

Middle diameter = 12 inches.
12

Sq. of midd. diam. = 144
Length = 15

2160
·418879 common multiplier.

25132740
418879
837758

904·778640

3. If the middle diameter of a parabolic spindle
and its length 9 feet, what is the solid content of th

Middle diam. = 3
3

Sq. of middle diam. = 9
Length = 9

81
Common multiplier ·418879

418879
3351032

Solidity = 33·929199 cubic feet.

I. Find the content of a parabolic spindle whose length is inches, and middle diameter 15 inches.

Middle diam. = 15
15

Sq. of midd. diam. = 225
Length = 28

6300
·418879

125663700
2513274

1728)2638·937700 cubic inches.
1·5271 cubic feet.

PROBLEM XXIX.

o determine the content of the middle frustum of a parabolic spindle.

Ex. 3. Determine the solid content of the middle frustum of arabolic spindle, its middle diameter being 32 inches, end neter 24 inches, and the length 40 inches.

Middle diameter = 32
32

64
96

1024
8

mes the sq. of midd. diam. = 8192
mes the sq. of less diam. = 1728
mes the prod. of diam. = 3072

12992
40 = length.

519680
Common multiplier = ·05236

3118080
155904
103936
259840

Solidity = 27210·44480 cubic inches.

3. Find the capacity of the middle frustum of a] spindle, its end diameter being 3 feet, middle diamete and length 7 feet.

Middle diameter =	5
	5
	25
	8
8 times the sq. of mid. diam. =	200
3 times the sq. of the less. =	27
4 times the prod. of diams. =	60
Sum =	287
	7 = length.
	2009
	·05236
	47124
	10472
Solidity =	105·19124 cubic feet

PROBLEM. XXX.

RULE I.

To find the solidity of an hyperboloid.

Ex. 2. The altitude of an hyperboloid is 50 inches, dius of the base 52, and the middle diameter 68 inch the content of the solid.

Radius of the base =	52
	52
	104
	260
	2704
(Middle diameter)^2 =	4624
	7328
	50 = height.
	366400
Common multiplier =	·5236
	2198400
	10992
	7328
	18320
Solidity =	191847·0400 cubic inches

3. If the altitude of an hyperboloid be 15 inches, the radiu f its base 16 inches, and the middle diameter 20 inches; wha the solidity?

Radius of the base =	16
	16
	96
	16
Sq. of rad. of the base =	256
Sq. of middle diam. =	400
	656
Altitude of the solid =	15
	3280
	656
	9840
	·5236
	209440
	41888
	47124
Solidity =	5152·2240 cubic inches.

RULE II.

find the solidity of an hyperboloid, supposing the major axis of the generating hyperbola to be given.

Ex. 2. The altitude of an hyperboloid is 40, the radius of it se 48, *and the* major axis of the generating hyperbola is 12 *d the content of the* solid.

$$
\begin{array}{rr}
\text{Diameter of the solid's base} = & 96 \\
 & 96 \\
\hline
 & 576 \\
 & 864 \\
\hline
 & 9216 \\
\text{Area of a circle to diam. } 1 = & \cdot 7854 \\
\hline
 & 36864 \\
 & 46080 \\
 & 73728 \\
 & 64512 \\
\hline
 & 7238\cdot 2464 \\
 & 20 = \tfrac{1}{2} \text{ altitude.} \\
\hline
\end{array}
$$

$\frac{1}{2}$ area of base $\times$ alt. $= 144764 \cdot 9280$

Hence $120 + 40 : 120 + 26\frac{2}{3} :: 144764 \cdot 928$: c

or, $160 : 146\frac{2}{3} :: 144764 \cdot 928$: content.

$80 : 73\frac{1}{3} :: 144764 \cdot 928$: content.

$$
\begin{array}{r}
73\tfrac{1}{3} \\
\hline
434294784 \\
1013354496 \\
\hline
10567839 \cdot 744 \\
48254 \cdot 976 \text{ for } \tfrac{1}{3} \\
\hline
80)10616094 \cdot 720 \\
\hline
\end{array}
$$

Solidity $=$ $132701 \cdot 184$ cubic inche

PROBLEM XXXI.

To find the solid content of the frustum of an hyperboi

Ex. 2. Find the solidity of the frustum of an hyperboloi *height is 12* inches; the diameter of the greater end be the diameter of the lesser end 6 inches; also the ·8·5.

greater $\frac{1}{2}$ diameter = 25
lesser $\frac{1}{2}$ diameter = 9
middle diameter = 72·25

$$\begin{array}{r} 106{\cdot}25 \\ 12 \end{array} = \text{altitude.}$$

$$\begin{array}{r} 1275{\cdot} \\ {\cdot}5236 \\ \hline 7650 \\ 3825 \\ 2550 \\ 6375 \\ \hline \end{array}$$

Solidity = 667·5900 cubic inches.

'he altitude of a hyperbolic frustum is 8 inches, the · diameter 14 inches, and lesser diameter 9 inches, also ıdle diameter 10·873 inches; required the content of stum.

greater semi-diam. = 49
lesser semi-diam. = 20·25
middle diam. = 118·222129

$$\begin{array}{r} 187{\cdot}472129 \\ 8 \end{array} = \text{altitude.}$$

$$\begin{array}{r} 1499{\cdot}777032 \\ {\cdot}5236 \\ \hline 8998662192 \\ 4499331096 \\ 2999554064 \\ 7498885160 \\ \hline \end{array}$$

Solidity = 785·2832539552 cubic inches.

4. The altitude of an hyperbolic frustum is 10 inches, greater diameter 26 inches, and the lesser diameter 14 inc also the middle diameter is 19·788 : determine the solid tent of the frustum.

Sq. of greater semi-diam. = 169
Sq. of lesser semi-diam. = 49
Sq. of middle diam. = 391·564944

609·564944
10 = altitude.

6095·64944
·5236

3657389664
1828694832
1219129888
3047824720

1728)3191·682046784

1·8473 cubic feet.

PROBLEM XXXII.

To determine the surface of a circular spindle whose lengt and breadth or middle diameter are given.

Ex. 2. Determine the superficial content of a circular spi whose length is 48 inches, and its middle diameter 30 incl

Here $24^2 + 15^2 = 576 + 225 = 801$
$\therefore \frac{801}{30} = 26\cdot7 =$ radius of the circle.
and $26\cdot7 - 15 = 11\cdot7 =$ central distance.

Now in order to find the length of the revolving arc (chord)^2 of half the arc ACB = 801 ∴ the chor $\sqrt{801} = 28\cdot302$.

$$\begin{array}{rl} 28{\cdot}302 & \\ 8 & \\ \hline 226{\cdot}416 & = 8 \text{ times the chord of } \tfrac{1}{2} \text{ arc.} \\ 48 \quad & = \text{chord of whole arc.} \\ \hline 3)178{\cdot}416 & \\ \hline 59{\cdot}472 & = \text{length of arc ACB, nearly.} \\ 11{\cdot}7 & = \text{central distance.} \\ \hline 416304 & \\ 654192 & \\ \hline 695{\cdot}8224 & = \text{arc} \times \text{central distance.} \\ 1281{\cdot}6 \quad & = \text{length of spindle} \times \text{rad. of arc.} \\ \hline 585{\cdot}7776 & \\ 6{\cdot}2832 & \\ \hline 11715552 & \\ 17573328 & \\ 46862208 & \\ 11715552 & \\ 35146656 & \\ \hline \end{array}$$

urface $= 3680{\cdot}55781632$ sq. inches.

ote. The arc A C B is more nearly equal to 59·6784 than to 59·472, for rule in Problem XII. is not a very near approximation, and if the ner value were used, the surface of the spindle would be equal to 5·3846 square inches, nearly.

. Find the superficial content of a circular spindle whose gth is 18 inches, and middle diameter 8 inches.

Here $9^2 + 4^2 = 81 + 16 = 97$

$\therefore \frac{97}{8} = 12{\cdot}125 =$ radius of the circle.

and $12{\cdot}125 - 4 = 1{\cdot}125 =$ central distance.

also $(\text{chord})^2$ of $\frac{1}{2}$ arc A C B $= 97$

$\therefore$ chord of $\frac{1}{2}$ arc ACB $= \sqrt{97} = 9{\cdot}849$

$$\begin{array}{r} 9{\cdot}849 \\ 8 \\ \hline 78{\cdot}792 \\ 18{\cdot} \\ \hline 3)60{\cdot}792 \\ \hline \end{array}$$

$$\begin{array}{rr} \text{Length of the arc ACB} = & 20{\cdot}264 \\ \text{Central dist.} = & 8{\cdot}125 \\ \hline & 101320 \\ & 40528 \\ & 20264 \\ & 162112 \\ \hline \text{Arc} \times \text{central distance} = & 164{\cdot}645000 \\ \text{Length of spindle} \times \text{rad.} = & 218{\cdot}25 \\ \hline & 53{\cdot}605 \\ & 6{\cdot}2832 \\ \hline & 107210 \\ & 160815 \\ & 428840 \\ & 107210 \\ & 321630 \\ \hline \text{Surface} = & 336{\cdot}8109360 \text{ sq. inches.} \end{array}$$

4. Determine the number of square inches contained in t surface of a circular spindle whose length is 3 feet, and m dle diameter 1 foot 6 inches.

Here $18^2 + 9^2 = 324 + 81 = 405$

and $\frac{405}{18} = 22{\cdot}5 =$ radius of the circle.

$\therefore 22{\cdot}5 - 9 = 13{\cdot}5 =$ central distance.

rd)^2 of $\frac{1}{2}$ arc ACB $= 405$

chord of $\frac{1}{2}$ arc A C B = 20·1221
8

imes chord of $\frac{1}{2}$ arc = 160·9768
ord of whole arc = 36

3)124·9768

gth of the arc A C B = 41·6589
Central distance = 13·5

2082945
1249767
416589

rc × central distance 562·39515
:h of spindle × radius 810·

247·60485
6·2832

49520970
74281455
198083880
49520970
148562910

Surface = 1555·750793520 sq. inches.

PROBLEM XXXIII.

d the superficial content of the middle frustum of a circula: spindle.

. 2. Find the superficial content of a wine pipe in the forn ircular spindle, whose length is 6 feet, bung diameter : ınd end diameter 2 feet.

Here $3^2 + (\frac{1}{2})^2 = \frac{37}{4}$

$\therefore \frac{37}{4} \div 1 = \frac{37}{4} = 9·25$ = radius of the circle.

and $9·25 - 1·5 = 7·75$ = central distance.

Now in order to find the length of the arc N C P, we h

$$(\text{chord})^2 \text{ of } \tfrac{1}{2} \text{ N C P} = \tfrac{37}{4} = 9\cdot25$$

$$\therefore \text{ chord of } \tfrac{1}{2} \text{ arc N C P} = 3\cdot04138$$

8

8 times chord of ½ arc = 24·33104
Chord of whole arc = 6

3)18·33104

Length of arc N C P = 6·11035, nearly.
Central distance = 7·75

3055175
4277245
4277245

Arc × central distance = 47·3552125
Length × radius = 55·5

8·1447875
6·2832

162895750
244343625
651583000
162895750
488687250

Surface = 51·17532882000 sq. feet.

3. If the length of the middle frustum of a circular be 25 inches, the middle diameter 13 inches, and end d 9 inches; what is its superficial content?

Here $(12\cdot5)^2 + 2^2 = 160\cdot25$

$\therefore \frac{160\cdot25}{4} = 40\cdot0625 =$ radius of the circle.

and $40\cdot0625 - 6\cdot5 = 33\cdot5625 =$ central distan

Likewise (chord)^2 of ½ arc N C P = 160·25

ıord of $\frac{1}{2}$ arc N C P = 12·66, nearly.

$$\begin{array}{r} 8 \\ \hline 101{\cdot}28 \\ 25 \\ \hline 3)\ 76{\cdot}28 \\ \hline \end{array}$$

Length of the arc = 25·42
33·5625

$$\begin{array}{r} 671250 \\ 1342500 \\ 1678125 \\ 671250 \\ \hline \end{array}$$

× cent. dist. = 853·158750
gth × radius = 1001·5625

$$\begin{array}{r} 148{\cdot}40375 \\ 6{\cdot}2832 \\ \hline 29680750 \\ 44521125 \\ 118723000 \\ 29680750 \\ 89042250 \\ \hline \end{array}$$

Surface = 932·450442000 sq. inches.

·. Determine the superficial content of the middle frustun ı circular spindle, whose length is 44 inches; the middl end diameters being 18 inches and 14 inches respectively

Here $22^2 + 2^2 = 488$

$\therefore \frac{488}{4} = 122$ = radius of the circle.

and $122 - 9 = 113$ = central distance.

Likewise (chord)^2 of $\frac{1}{2}$ arc N C P = 488

$\therefore$ chord $\frac{1}{2}$ arc $= 22{\cdot}0907$

$$\begin{array}{r}
22{\cdot}0907 \\
8 \\
\hline
176{\cdot}7256 \\
44 \\
\hline
3)132{\cdot}7256 \\
\hline
\end{array}$$

Length of arc NCP $= 44{\cdot}2419$

$$\begin{array}{rl}
44{\cdot}2419 & \\
113 & = \text{central dist.} \\
\hline
1327257 & \\
4866609 & \\
\hline
\end{array}$$

Arc $\times$ cent. dist. $= 4999{\cdot}3347$
Length $\times$ radius $= 5368{\cdot}$

$$\begin{array}{r}
368{\cdot}6653 \\
6{\cdot}2832 \\
\hline
7373306 \\
11059959 \\
29493224 \\
7373306 \\
22119918 \\
\hline
\end{array}$$

Surface $= 2316{\cdot}39781296$ sq. inches.

5. What is the superficial content of the middle frust a circular spindle, whose length, middle, and end diai are 100, 50, and 30 ?

Here $50^2 + 10^2 = 2600$
$\therefore \frac{2600}{20} = 130 =$ radius of the circle.
Hence $130 - 25 = 105 =$ central distance.

Also (chord)^2 of $\frac{1}{2}$ arc NCP $= 2600$

$$
\begin{array}{lr}
\therefore \text{ chord of } \tfrac{1}{2} \text{ arc NCP} = & 50{\cdot}9901 \\
 & 8 \\ \hline
8 \text{ times chord of } \tfrac{1}{2} \text{ arc} = & 407{\cdot}9208 \\
\text{Chord of whole arc} = & 100{\cdot} \\ \hline
 & 3)307{\cdot}9208 \\ \hline
\text{Length of arc NCP} = & 102{\cdot}6402 \\
\text{Central distance} = & 105 \\ \hline
 & 5132010 \\
 & 1026402 \\ \hline
\text{Arc} \times \text{central dist.} = & 10777{\cdot}2210 \\
\text{Length} \times \text{radius} = & 13000\phantom{{\cdot}2210} \\ \hline
 & 2222{\cdot}779 \\
 & 6{\cdot}2832 \\ \hline
 & 4445558 \\
 & 6668337 \\
 & 17782232 \\
 & 4445558 \\
 & 13336674 \\ \hline
\text{Surface} = & 13966{\cdot}1650128 \text{ sq. inches.}
\end{array}
$$

OF THE REGULAR BODIES.

PROBLEM I.

To determine the solid content of a tetrahedron.

Ex. 2. *Find the solid content of a tetrahedron whose side* ...
ıches.

Tabular multiplier = ·11785 = content of a tetrahedron [sid
27

82495
23570

Solidity = 3·18195 cubic inches.

3. Determine the solidity of a tetrahedron whose lin
is 6 feet.

Tabular multiplier = ·11785
216 = 6^3

70710
11785
23570

Solidity = 25·4556 cubic feet.

4. If the side of a tetrahedron be $4\frac{1}{2}$ inches; deter
content of the solid.

Tabular multiplier = ·11785
91·125 = $(4·5)^3$

58925
23570
11785
11785
106065

Solidity = 10·73908125 cubic inches.

PROBLEM II.

To determine the superficial content of a tetrahed

Ex. 2. Find the surface of a tetrahedron, whose
is r ·

ılar multiplier $= 1{\cdot}73205 =$ surface of a tetrahedron whose [side is unity.

$$\begin{array}{r} 25 \\ \hline 866025 \\ 346410 \\ \hline \end{array}$$

Surface $= 43{\cdot}30125$ sq. inches.

. Find the surface of a tetrahedron whose linear side is 7 ıes.

Tab. multiplier $= 1{\cdot}73205$

$$\begin{array}{r} 49 = 7^2 \\ \hline 1558845 \\ 692820 \\ \hline \end{array}$$

Surface $= 84{\cdot}87045$ sq. inches.

f $R =$ the radius of the circumscribed sphere, $\varrho =$ that of inscribed sphere, $\lambda =$ a linear side of the solid, $\varphi =$ the ole surface of ditto, and $s =$ the solid content, we can easily uce the following equations:

$$\varrho = \tfrac{1}{12}\,\lambda\sqrt{6}$$

$$R = \tfrac{1}{4}\,\lambda\sqrt{6}$$

$$\lambda = (\tfrac{1}{3}\,\varphi\sqrt{3})^{\frac{1}{2}} = (6\,s\sqrt{2})^{\frac{1}{3}} = \tfrac{2}{3}\,R\sqrt{6} = 2\,\varrho\sqrt{6}$$

$$\varphi = \lambda^2\sqrt{3} = 6(s^2\sqrt{3})^{\frac{1}{3}} = \tfrac{8}{3}\,R^2\sqrt{3} = 24\,\varrho^2\sqrt{3}$$

$$s = \tfrac{1}{2}\,\lambda^3\sqrt{2} = \tfrac{1}{36}\,\varphi\,(2\,\varphi\sqrt{3})^{\frac{1}{2}} = \tfrac{8}{27}\,R^3\sqrt{3} = 8\,\varrho^3\sqrt{3}$$

$$R = \tfrac{1}{4}\,\lambda\sqrt{6} = \tfrac{1}{4}\,(2\,\varphi\sqrt{3})^{\frac{1}{2}} = \tfrac{3}{2}\,(\tfrac{1}{3}\,s\sqrt{3})^{\frac{1}{3}} = 3\,\varrho$$

$$\varrho = \tfrac{1}{12}\,\lambda\sqrt{6} = \tfrac{1}{12}\,(2\,\varphi\sqrt{3})^{\frac{1}{2}} = \tfrac{1}{2}\,(\tfrac{1}{3}\,s\sqrt{3})^{\frac{1}{3}} = \tfrac{1}{3}\,R.$$

With regard to the hexahedron or cube, the diameter of the ıcribed sphere is evidently equal to a linear side of the cube; d the diameter of the circumscribed sphere is equal to a diaınal of the cube; hence, retaining the same notation, we shall ıtain the following equations:

1. $\lambda = \sqrt{\tfrac{1}{6}\,\varphi} = \sqrt[3]{s} = \tfrac{2}{3}\,R\sqrt{3} = 2\,\varrho$
2. $\varphi = 6\,\lambda^2 = 6\,\sqrt[3]{s^2} = 8\,R^2 = 24\,\varrho^2$
3. $s = \lambda^3 = \tfrac{1}{6}\,\varphi\sqrt{\tfrac{1}{6}\varphi} = \tfrac{8}{9}\,R^3\sqrt{3} = 8\,\varrho^3$
4. $R = \tfrac{1}{2}\lambda\sqrt{3} = \tfrac{1}{2}\sqrt{\tfrac{1}{2}\varphi} = \tfrac{1}{3}\sqrt{3}\,s^{\frac{1}{3}} = \varrho\sqrt{3}$
5. $\varrho = \tfrac{1}{2}\lambda = \tfrac{1}{2}\sqrt{\tfrac{1}{6}\,\varphi} = \tfrac{1}{2}\sqrt[3]{s} = \tfrac{1}{3}\,R\sqrt{3}.$

PROBLEM III.

To find the solidity of an octahedron.

Ex. 2. Determine the solid content of an octahe
linear side is 4 inches.

Tab. multiplier = ·4714 = content of an oct. w

$$
\begin{array}{r}
64 = 4^3 \\
\hline
18856 \\
28284 \\
\hline
\end{array}
$$

Content = 30·1696 cubic inches.

3. If the linear side of an octahedron be 8 feet
the content of the solid.

Tab. multiplier = ·4714

$$
\begin{array}{r}
512 = 8^3 \\
\hline
9428 \\
4714 \\
23570 \\
\hline
\end{array}
$$

Content = 241.3568 cubic inches.

PROBLEM IV.

To find the surface of an octahedron.

Ex. 2. Find the superficial content of an octahe
inear side is 3 inches.

Tab. multiplier = 3·4641

$$9 = 3^2$$

Surface = 31·1769 sq. inches.

termine the superficial content of an octah
is 4 inches.

Tab. multiplier = 3·4641
$\quad 16 = 4^2$

207846
34641

Surface = 55·4256 sq. inches.

4. If the linear side of an octahedron be $2\frac{1}{2}$ feet, what is the superficial content?

Tab. multiplier = 3·4641
$\quad 6·25 = (2·5)^2$

173205
69282
207846

Surface = 21·650625 sq. feet.

The following equations, expressing the relation between the several dimensions of the octahedron, are easily reduced.

$$\lambda = (\tfrac{1}{6}\,\varphi\,\sqrt{3})^{\frac{1}{2}} = (\tfrac{3}{2}\,s\,\sqrt{2})^{\frac{1}{3}} = R\,\sqrt{2} = \varrho\,\sqrt{6}$$

$$\varphi = 2\,\lambda^2\sqrt{3} = 6\,(\tfrac{1}{2}\,s^2\sqrt{3})^{\frac{1}{3}} = 4\,R^2\sqrt{3} = 12\varrho^2\sqrt{3}$$

$$s = \tfrac{1}{3}\,\lambda^3\,\sqrt{2} = \frac{\varphi}{18}(\varphi\,\sqrt{3})^{\frac{1}{2}} = \tfrac{4}{3}\,R^3 = 4\,\varrho^3\,\sqrt{3}$$

$$R = \tfrac{1}{2}\,\lambda\,\sqrt{2} = \left(\frac{\varphi}{12}\sqrt{3}\right)^{\frac{1}{2}} = (\tfrac{3}{4}\,s)^{\frac{1}{3}} = \varrho\,\sqrt{3}$$

$$\varrho = \tfrac{1}{6}\,\lambda\,\sqrt{6} = \tfrac{1}{6}\,(\varphi\sqrt{3})^{\frac{1}{2}} = \left(\frac{s}{12}\sqrt{3}\right)^{\frac{1}{3}} = \tfrac{1}{3}\,R\,\sqrt{3}$$

PROBLEM V.

To determine the solidity of a dodecahedron.

Ex. 1. If the linear side of a dodecahedron be 1 foot in length, what is its solidity?

Tab. multiplier = 7.66312
1 = 1^3

Content = 7·66312 cubic feet.

3. The linear side of a dodecahedron is 3 feet; det the content of the solid.

Tab. multiplier = 7·66312
27 = 3^3

5364184
1532624

Content = 206·90424 cubic feet.

4. A dodecahedron has its linear side equal to $2\frac{1}{2}$ in find the content of the solid.

Tab. multiplier = 7·66312
15·625 = $(2{\cdot}5)^3$

3831560
1532624
4597872
3831560
766312

Solidity = 119·73625000 cubic inches.

PROBLEM VI.

To find the superficial content of a dodecahedro

E. 2. The linear side of a dodecahedron is 12 inch perficial content of the solid?

Tab. multiplier = 20·64573
169 = 13^2

18581157
12387438
2064573

144)3489·12837 sq. inches.

24·2335 sq. feet.

If the linear side of a dodecahedron be $2\frac{1}{2}$ feet, what is uperficial content of the solid?

Tab. multiplier = 20·64573
6·25 = $(2·5)^2$

10322865
4129146
12387438

Surface = 129·0358125 sq. feet.

taining the former notation, we shall find, that

$$= \left\{\frac{\varphi}{15}(5-2\sqrt{5})^{\frac{1}{2}}\right\}^{\frac{1}{2}} = \left\{\frac{s}{5}(470-210\sqrt{5})^{\frac{1}{2}}\right\}^{\frac{1}{3}}$$

$$= \frac{R}{3}(\sqrt{15}-\sqrt{3}) = \varrho\,\{50-22\sqrt{5}\}^{\frac{1}{2}}$$

$$= 15\,\lambda^2\{\tfrac{1}{5}(5+2\sqrt{5})\}^{\frac{1}{2}} = 3\{10\,s^2(130-58\sqrt{5})^{\frac{1}{2}}\}^{\frac{1}{3}}$$

$$= 10\,R^2\{\tfrac{1}{5}(10-2\sqrt{5})\}^{\frac{1}{2}} = 30\,\varrho^2(130-58\sqrt{5})^{\frac{1}{2}}$$

$$= 5\,\lambda^2\{\tfrac{1}{40}(47+21\sqrt{5})\}^{\frac{1}{2}} = \frac{\varphi}{30}\left\{\frac{\varphi}{6}(650+290\sqrt{5})^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$

$$= \frac{20R^3}{3}\{\tfrac{1}{30}(3+\sqrt{5})\}^{\frac{1}{2}} = 10\,\varrho^3\{130-58\sqrt{5}\}^{\frac{1}{2}}$$

$$= \frac{\lambda}{4}\{\sqrt{15}+\sqrt{3}\} = \left\{\frac{\varphi}{40}(10+2\sqrt{5})^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{3s}{40}(90-30\sqrt{5})^{\frac{1}{2}}\right\}^{\frac{1}{3}} = \varrho\{15-6\sqrt{5}\}^{\frac{1}{2}}$$

$$= \frac{\lambda}{20}\{250+110\sqrt{5}\}^{\frac{1}{2}} = \tfrac{1}{20}\left\{\frac{2\varphi}{3}(650+290\sqrt{5})^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{s}{200}(650+290\sqrt{5})^{\frac{1}{2}}\right\}^{\frac{1}{3}} = R\,\{\tfrac{1}{15}(5+2\sqrt{5})\}^{\frac{1}{2}}.$$

PROBLEM VII.

To determine the solidity of an icosahedron.

Ex. 2. If the linear side of an icosahedron be 3 inches, the content of the solid.

Tab. multiplier $= 2\cdot18169$
$27 = 3^3$

1527183
436338

Content $= 58\cdot90563$ cub. inches.

3. Find the content of an icosahedron, whose linear si one foot.

Tab. multiplier $= 2\cdot18169$
$1 = 1^3$

Content $= 2\cdot18169$ cubic feet.

PROBLEM VIII.

To find the superficial content of an icosahedron.

Ex. 2. Find the superficial content of an icosahedron linear side is 3 inches.

Tabular multiplier $8\cdot66025$
$9 = 3^2$

Surface $= 77\cdot94225$ square inches.

3. If the linear side of an icosahedron be 4 inches, w its superficial content?

Tabular multiplier $8\cdot66025$
$16 = 4^2$

5196150
866025

Surface $= 138\cdot56400$ square inches,

ıe same notation being retained, we shall find with regard .e icosahedron, that

$$= \left(\frac{\varphi}{15}\sqrt{3}\right)^{\frac{1}{2}} = \left\{\frac{6\,s}{5}\sqrt{\tfrac{1}{2}(7-3\sqrt{5})}\right\}^{\frac{1}{3}}$$
$$= R\sqrt{\tfrac{1}{5}(10-2\sqrt{5})} = \varrho\,(42-18\sqrt{5})^{\frac{1}{2}}$$
$$= 5\,\lambda^2\sqrt{3} = 3\,\{s^2(70\sqrt{3}-30\sqrt{15})\}^{\frac{1}{3}}$$
$$= 2\,R^2\,\{5\sqrt{3}-\sqrt{15}\} = 3\,\varrho^2\,\{7\sqrt{3}-3\sqrt{15}\}$$
$$= \frac{5\,\lambda^3}{6}\{\tfrac{1}{2}(7+3\sqrt{5})\}^{\frac{1}{2}} = \frac{\varphi}{18}\{\tfrac{1}{10}(7\sqrt{3}+3\sqrt{15})\}^{\frac{1}{2}}$$
$$= \frac{2\,R^3}{3}\{10+2\sqrt{5}\}^{\frac{1}{2}} = 10\,\varrho^3\,\{7\sqrt{3}-3\sqrt{15}\}$$
$$= \frac{\lambda}{2}\{\tfrac{1}{2}(5+\sqrt{5})\}^{\frac{1}{2}} = \frac{\varphi}{2}\{\tfrac{1}{30}(5\sqrt{3}+\sqrt{15})\}^{\frac{1}{2}}$$
$$= \left\{\frac{3\,s}{4}\{\tfrac{1}{10}(5-\sqrt{5})\}^{\frac{1}{2}}\right\}^{\frac{1}{3}} = \varrho\,\{15-6\sqrt{5}\}$$
$$= \frac{\lambda}{2}\{\tfrac{1}{6}(7+3\sqrt{5})\}^{\frac{1}{2}} = \frac{\varphi}{6}\{\tfrac{1}{10}(7\sqrt{3}+3\sqrt{15})\}^{\frac{1}{2}}$$
$$= \frac{s}{2}\{\tfrac{1}{30}(7\sqrt{3}+3\sqrt{15})\}^{\frac{1}{3}} = R\,\{\tfrac{1}{15}(5+2\sqrt{5})\}^{\frac{1}{2}}$$

ON CYLINDRICAL RINGS.

PROBLEM I.

To find the convex surface of a cylindric ring.

:. 2. The inner diameter of a ring is 16 inches, and its ness 4 inches; what is its superficial content?

$$\begin{array}{rl}
16 & = \text{inner diam.} \\
4 & = \text{thickness.} \\
\hline
20 & \\
4 & = \text{thickness.} \\
\hline
80 & \\
9{\cdot}8696 & \\
\hline
\end{array}$$

Surface = 789·5680 square inches.

The thickness of a ring is 3 inches, and its inner dia- 1 foot; what is its superficial content?

$$
\begin{array}{rl}
12 & = \text{inner diam.} \\
3 & = \text{thickness.} \\
\hline
15 & \\
3 & = \text{thickness.} \\
\hline
45 & \\
9\cdot8696 & \\
\hline
493480 & \\
394784 & \\
\hline
\text{Surface} = 444\cdot1320 & \text{square inches.}
\end{array}
$$

4. The thickness of a ring is 4 inches, and its inn[er diam-]eter 18 inches; what is its convex superficies?

$$
\begin{array}{rl}
18 & = \text{inner diam.} \\
4 & = \text{thickness.} \\
\hline
22 & \\
4 & = \text{thickness.} \\
\hline
88 & \\
9\cdot8696 & \\
\hline
789568 & \\
789568 & \\
\hline
\text{Surface} = 868\cdot5248 & \text{square inches.}
\end{array}
$$

5. If the thickness of a ring be 2 inches, and diameter 18 inches; what is its convex superficies?

$$
\begin{array}{rl}
18 & = \text{inner diam.} \\
2 & = \text{thickness.} \\
\hline
20 & \\
2 & = \text{thickness.} \\
\hline
40 & \\
9\cdot8696 & \\
\hline
\text{Surface} = 394\cdot7840 & \text{square inches.}
\end{array}
$$

PROBLEM II.

To find the solid content of a cylindric ring.

2. Find the solidity of a cylindrical ring whose thickn
ches, and inner diameter 8 inches.

$$\begin{array}{r l}
8 & = \text{inner diameter.} \\
3 & = \text{thickness.} \\
\hline
11 & \\
9 & = 3^2 = (\text{thickness})^2 \\
\hline
99 & \\
2{\cdot}4674 & \\
\hline
222066 & \\
222066 & \\
\hline
\text{Content} = 244{\cdot}2726 & \text{cubic inches.}
\end{array}$$

f the inner diameter of a cylindric ring be 18 inches, a
kness 4 inches: what is the solid content?

$$\begin{array}{r l}
18 & = \text{inner diameter.} \\
4 & = \text{thickness.} \\
\hline
22 & \\
16 & = 4^2 = (\text{thickness})^2 \\
\hline
352 & \\
2{\cdot}4674 & \\
\hline
49348 & \\
123370 & \\
74022 & \\
\hline
\text{Content} = 868{\cdot}5248 & \text{cubic inches.}
\end{array}$$

4. Required the solidity of a cylindric ring whose thi
is 2 inches, and inner diameter 12 inches.

$$\begin{array}{rl}
12 & = \text{inner diameter.} \\
2 & = \text{thickness,} \\
\hline
14 & \\
4 & = 2^2 = (\text{thickness})^2 \\
\hline
56 & \\
2{\cdot}4674 & \\
\hline
148044 & \\
123370 & \\
\hline
\text{Solidity} = 138{\cdot}1744 & \text{cubic inches.}
\end{array}$$

5. Determine the solidity of a cylindric ring whose t
is 4 inches and inner diameter 26 inches.

$$\begin{array}{rl}
26 & = \text{inner diameter.} \\
4 & = \text{thickness.} \\
\hline
30 & \\
16 & = 4^2 = (\text{thickness})^2 \\
\hline
480 & \\
2{\cdot}4674 & \\
\hline
1973920 & \\
98696 & \\
\hline
\text{Solidity} = 1184{\cdot}3520 & \text{cubic inches.}
\end{array}$$

6. If the thickness of a cylindrical ring be 2¼ in
its inner diameter 14½ inches; what is its solid con

$$\begin{array}{r l}
14{\cdot}25 & = \text{inner diameter.} \\
2{\cdot}5 & = \text{thickness.} \\
\hline
16{\cdot}75 & \\
6{\cdot}25 & = (2{\cdot}5)^2 = (\text{thickness})^2 \\
\hline
8375 & \\
3350 & \\
10050 & \\
\hline
104{\cdot}6875 & \\
2{\cdot}4674 & \\
\hline
4187500 & \\
7328125 & \\
6281250 & \\
4187500 & \\
2093750 & \\
\hline
\end{array}$$

Content = 258·30593750 cubic inches.

ARTIFICER'S AND GLAZIER'S WORK.

:. 2. If a pane of glass be 2 feet $8\frac{3}{4}$ inches long, and 1 ŀ inches broad; how many square feet does it contain?

	ft.	*in.*	*pts.*
Length =	2	8	9
Breadth =	1	4	
	2	8	9
		10	11
Surface required =	3	7	8

Determine the cost of glazing a triangular window at ›er foot, supposing the base to be 6 feet 6 inches, ar rpendicular height 4 feet 9 inches.

ft.	*in.*		
6	6 = base.		
4	9 = altitude.		
26	0		
4	10	6	
30	10	6 = area of window according to the	
3*d*. is $\frac{1}{4}$ 7	8	7	6
20) 38	7	1	6
Ans. £ 1	18	7 nearly.	

4. There is a house having 3 rows of windows windows in each row or tier, also the breadth of each is 3 feet 11 inches; required the expense of glazing at 14*d*. per square foot, supposing the length of a in the

1st tier to be 7 *ft*. 10 *in*.
2d tier to be 6 8
3d tier to be 5 4.

	ft. *in*.	*ft*. *in*.
3 ×	7 10 =	23 6
3 ×	6 8 =	20 0
3 ×	5 4 =	16 0

Sum of the *lengths* of all the windows	59	6	
	3	11 = common br	
	178	6	
	54	6	6
Area of all the windows =	233	0	6
2*d*. is $\frac{1}{6}$	38	10	1
	20)271	10	7

Answer is £13 11 10$\frac{1}{2}$ nearly.

5. There is a house having three tiers of window, three in a er; the length of a window in the

lowest tier is 7 *ft.* 9 *in.*
middle tier 6 6
highest tier 5 $3\frac{1}{4}$;

lso over the door a circular window whose diameter is 1 foot $0\frac{1}{4}$ inches; determine the charge of glazing these windows at 3*d.* per sq. foot, their common breadth being 3 feet 9 inches.

	feet. *in.*	*feet.* *in.*		
	3×7 9 =	23 3		
	3×6 6 =	19 6		
	3×5 $3\frac{1}{4}$ —	15 $9\frac{3}{4}$		
Sum of all the lengths. =		58	6	9
Common breadth =		3	9	
		175	8	3
		43	11	1 7
Area of all the windows =		219	7	4 7
Area of circular window =		3	6	
		223	1	4 9
	1*d.* is $\frac{1}{12}$	18	7	1 4
	20)	241	8	6 1
	Answer	£12	1	$8\frac{1}{2}$

6. There is a house having 4 tier of windows; 4 in each tier: the length of a window in the

lowest tier is 7 *ft.* 4 *in.*
2d tier 6 0
3d tier 5 6
4th tier 4 0

What will be the expense of glazing these windows at 1s. 6d *per foot, their* common breadth being as in the last example?

feet.	in.		feet.	in.
4 × 7	4	=	29	4
4 × 6	0	=	24	0
4 × 5	6	=	22	0
4 × 4	0	=	16	0

Sum of all the lengths = 91 4
Common breadth = 3 9

274 0
68 6 0

342 6
6*d*. is ½ 171 3.

20)513 9

£25 13 9 answer.

PAINTER'S WORK.

Ex. 2. How many square yards are contained in of a room whose height is 16 feet 6 inches, and co feet 9 inches?

feet.	in.	
97	9	
16	6	
1564	0	
48	10	6
9)1612	10	6
Ans. 179 1	10	6

By Decimals.

97·75
16·5
48875
58650
9775
9)1612·875
179·208 sq. yards. The answer.

he height of a room is 14 feet 10 inches, and the cir-
ence or compass 21 feet 8 inches; how many square
oes it contain?

feet.	*in.*	
21	8	
14	10	
303	4	
18	0	8
9)321	4	8
Ans. 35	6 nearly.	

low many square yards of painting are there in a room
is 12 feet 4 inches high, and 65 feet 6 inches in compass?

feet.	*in.*	
65	6	
12	4	
786	0	
21	10	0
9)807	10	
Ans. 89	6	

5. A glazier painted a room at 8*d* per yard. The
are as follow:

The height of the room is	11 *ft.*	7 *in.*		
The girt or compass	74	10		
The door	7	6 by	3 *ft.*	9 *in.*
Five window shutters, each	6	8 by	3	4
The breaks in the windows	0	14 by	8	0
The chimney	6	9 by	5	0
A closet	3	6 by	11	7 with
front; with shelving	22	6 by	0	10; wha

cost, supposing the shutters, doors, and shelves to
on both sides?

feet.	*in.*	
6	9	
5	0	
33	9	= chimney spac

feet.	*in.*		
74	10		
11	7		
823	2		
43	7	10	
866	9	10	
33	9		to be deducted.
833	0	10	= area of the room.

feet.	*in.*		
7	6		= door's length.
3	9		= breadth.
22	6		
5	7	6	
28	1	6	= area of

	feet.	in.	
	6	8	
	3	4	
	20	0	
	2	2	8
ea of shutter =	22	2	8
			5
f 5 shutters =	111	1	4

	feet.	in.	
of the break is =	8	0	
Width =	3	4	
	11	4	
		10	
	113	4	
h of break =	1	2	
	113	4	
	18	10	8
the breaks =	132	2	8

	feet.	in.	
Closet's depth =	3	6	
Frontage =	4	9	
	8	3	
		2	
	16	6	
	11	7 height.	
	181	6	
	9	7	6
ea of closet =	191	1	6

feet.	in.		
22	6		= length of a shelf.
	2		
45	0		
0	10		= breadth.
37	6		= area of shelves.
833	0	10	= area of the ro
28	1	6	= area of door.
111	1	4	= area of the sh
132	2	8	= area of the br
191	1	6	= inside area of
9)1333	1	10	
148	1	6	

Now 6*d*. is ½ 1*s*.	74	0	9
2*d*. is ⅓ 6*d*.	24	8	3
	20)98	9	0
Ans.	£4	18	9

PAVIOUR'S WORK.

Ex. 2. What is the cost of paving a rectangular foot 3*s*. 4*d*. per yard; the length being 35 feet 4 inches, and 3 feet 3 inches?

	feet.	in.	
	35	4	
	3	3	
	106	0	
	8	10	
	9)114	10	
	12	9	1 &c.
			3
Cost at 3*s*. per yard	38	3	3
4*d*. is ⅓ of 1*s*.	4	3	0
	20)42	6	3
	£ 2	2	6 the answer.

ı rectangular court yard is 42 feet 9 inches long, and 6 inches wide; also a footway goes quite through it, 5 feet 6 inches in breadth; the footway is laid with stone 5*d.* per yard, and the rest with pebbles at 3*s.* per yard: rill the whole come to?

	feet.	in.		
Length of path =	68	6		
Breadth =	5	6		
	342	6		
	34	3		
	9)376	9		
	41	10	4	= area of path.
			3	
	125	7	0	
6*d.* is $\frac{1}{2}$	20	11	2	
	20)146	6	2	
	£ 7	6	6	= cost of paving the path with stone.

	feet.	in.		
Width of court =	68	6		
Length =	42	9		
	2877	0		
	51	4	6	
	9)2928	4	6	
Area of the court =	325	4	6	
	41	10	4	
·ea of pebbled part =	283	6	2	
			3	
	20)850	6	6	
	42	10	$6\frac{1}{2}$	
	7	6	6	
	£49	17	$0\frac{1}{2}$	= cost required.

4. Find the expense of paving a rectangular court-yard, length is 63 feet, and breadth 45 feet; supposing that it tains an unpaved walk in the form of a circular ring breadth is $2\frac{1}{2}$ feet, and inner diameter 19 feet: the rate of ing being 3*s*. a square yard.

By Rule II. in Problem XIV. for finding the area of cular ring, we have

$$
\begin{array}{rl}
24 & = \text{sum of diameters.} \\
14 & = \text{difference of diameters.} \\
\hline
336 & \\
\cdot 7854 & \\
\hline
47125 & \\
23562 & \\
23562 & \\
\hline
263\cdot 8944 & = \text{area of circular path.} \\
\hline
\end{array}
$$

$$
\begin{array}{rl}
63 & = \text{length.} \\
45 & = \text{breadth.} \\
\hline
315 & \\
252 & \\
\hline
2835 & = \text{area of the yard.} \\
263\cdot 8944 & \\
\hline
9)2571\cdot 1056 & = \text{area of paved p} \\
\hline
285\cdot 6784 & \text{sq. yards.} \\
3 & \\
\hline
857\cdot 0352 & \text{shillings.} \\
12 & \\
\hline
\cdot 4224 & \text{pence.} \\
4 & \\
\hline
1\cdot 6896 & \\
\hline
\end{array}
$$

Ans. £42 1

PLUMBER'S WORK.

A gentleman wishes to have the roof of his house covered
ead which will cost 19*s.* per cwt. The length of the
s 43 feet, and breadth or girt over it 32 feet; also the
ing is 60 feet long and 2 feet wide: what will be the
upposing the roof to contain 9 lb. to the square foot, and
tter to contain only 8 lb. to the square foot?

$$\begin{array}{r l}
43 & = \text{length of the roof.} \\
32 & = \text{girt over it.} \\
\hline
86 & \\
129 & \\
\hline
1376 & = \text{no. of sq. feet in the roof.} \\
\text{and } 120 & = \text{no. of sq. feet in the gutters.} \\
\text{Also } 1376 \times 9 & = 12384 \text{ lbs. of lead.} \\
120 \times 8 & = 960 \text{ lbs.}
\end{array}$$

$$\begin{array}{r l}
112)13344 & \\
\hline
119{\cdot}14285 & \text{cwts. of lead.} \\
19 & \\
\hline
107228565 & \\
11914285 & \\
\hline
2263{\cdot}71415 & \text{shillings.} \\
12 & \\
\hline
8{\cdot}56980 & \\
4 & \\
\hline
3{\cdot}27920 & \\
\hline
\end{array}$$

Ans. £113. 3*s.* 8½*d.*

What will be the expense of a lead covering for the roof
choir of York cathedral; supposing the length of the
be 210 feet, and the girt over it 80 feet; also the gut-
to be 4 feet wide, of the same length as the roof, and to
9 lbs. to the square foot: the roof containing only at
of 8 lbs. per square foot?

```
                         210
                           4
                         ---
                         840 = area of gutter.
                           9
                        ----
                        7560 lbs. of lead in gutter.
                        ----
       Length of roof = 210 feet.
         Girt over it =  80
         Area of roof = 16800 sq. feet.
                            8
No. of lbs. in the roof = 134400
No. of lbs. in the gut. =   7560
                         ------
                   112)141960
                        ------
                        1267·5 cwt. of lead.
                           19s. = price of 1 cwt.
                        ------
                        114075
                        12675
                        ------
                        24082·5 = 24082s. 6d.
                              20)-----------
                                £1204 2 6 Ans.
```

CARPENTER'S AND JOINER'S WORK.

Ex. 2. A floor is 53 feet 6 inches long, and 47 feet 9 broad: how many squares will it contain?

```
       feet. in.
        53   6 = length   = 53·5  feet.
        47   9 = breadth  = 47·75 feet.
                            -----
                            23875
                           14325
                          23875
                     100)2554·625
                          -------
                            25 54
                          -------
                           Ans. 25 squares.
```

partition is 91 feet 9 inches long, and 11 feet 3 inches how many squares will it contain?

	feet.	in.	
ngth =	91	9	= 91·75 feet.
eadth =	11	3	= 11·25 feet.

```
           45875
          18350
          9175
         9175
     100)1032·1875
          10 squares, 32 feet; the answer.
```

a house within the walls be 44 feet 6 inches long, and t 3 inches broad; how many squares of roofing will ?

feet.	in.	
18	3	= 18·25 feet = breadth.
		9·125 = $\frac{1}{2}$ breadth.

```
        27·375 = girt over the roof.
          44·5 = length of the roof.
        136875
       109500
      109500
  100)1218·1875
        12 squares, 18 feet; the answer.
```

f a house measure within the walls 52 feet 8 inches in , and 30 feet 6 inches in breadth, and the roof be of a ch: what will it cost roofing at 10s. 6d. per square?

feet.	in.	
30	6	= breadth.
15	3	= ½ breadth.
45	9	= girt over the roof.
52	8	= length of the roof.
2379	0	
30	6	
2409	6	
	10	
24095	0	
6*d*. is ½ 1204	9	
100)25299	9	
252·99		
12		
11·88		
4		
3·52		*Ans.* £12 1

6. What will be the cost of wainscotting a room per square yard; the height, including the cornice an ings, being 12 feet 6 inches, and the circuit of the ro 83 feet 8 inches. Also there are three window-shutt 7 feet 8 inches, by 3 feet 6 inches, and the door 7 f feet 6 inches; the door and shutters which are worked sides, being reckoned work and a half?

feet.	in.	
7.	8	
3	6	
23	0	
3	10	
26	10	
	3	
80	6	
40	3	being work and a
120	9	

feet.	*in.*	
3	6	
7	0	
24	6	
12	3	being work and half.
36	9	

	feet.	*in.*			
	12	6	= height of room.		
	83	8	= circuit.		
	1037	6			
	8	4			
	1045	10			
ted on account vs and door.	80	6			
	24	6			
room as single ork.	940	10			
	120	9			
	36	9			
	9)1098	4			
Sq. yards	122	0	5	4	
				5	
Price at 5*s*.	610	2	2	8	
6*d* is $\frac{1}{2}$	61	0	0	10	
	20) 671	2	3	6	
	£33	11	2	answer required.	

BRICKLAYER'S WORK.

Ex. 2. How many rods of standard brick-work a
a wall whose length is 57 feet 3 inches, and heigh
inches; the wall being $2\frac{1}{2}$ bricks thick?

	feet.	in.	
Length =	57	3	
Height =	24	6	
	1374	0	
	28	7	6
	1402	7	6 = superficial content
			5 = no. of $\frac{1}{2}$ bricks in th
	3)7013	1	6
	*272)2337	8	6

Ans. 8 rods, 17 sq. yards, 6 sq. feet, 8

3. Required the content of a wall which is 62 fe
long, and 14 feet 8 inches high, the thickness bein

	feet.	in.	
	62	6	
	14	8	
	875	0	
	41	8	
	916	8	
		5	
3)	4583	4	
272)	1527	9	4

Ans. 5 rods, 167 feet, 9 in

* See Table to ARTIFICER'S WORK, in the Mens

. If a triangular gable be raised 17 feet 6 inches on the wall of a house; what is the content, supposing the thicks of the gable to be 2 bricks, and its base 24 feet 9 inches?

	feet.	in.		
	24	9		= breadth of gable.
	17	6		= altitude of ditto.
	420	9		
	12	4	6	
2)	433	1	6	
	216	6	9	= surface of the gable.
			4	= no. of ½ bricks.
3)	866	3	0	
272)	288	9		

Ans. 1 rod, 16 feet, 9 parts.

. There is a building whose side wall is 45 feet long on the side, and end wall 15 feet broad on the inside; the height he building is 20 feet, and the gable at each end of the l is 6 feet high; what is the true content of the whole in dard rods, the walls being uniformly 2 bricks thick?

Breadth of gable =	15 feet.
	6
Surface of the two gables =	90
Length of building =	45 feet.
Breadth =	15
	60
	2
Compass of the building =	120
Height =	20
Surface of the walls =	2400
	90
	2490
No. of ½ bricks thick =	4
3)	9960
272)	3320 sq. feet.
	12·205 rods.

MASON'S WORK.

Ex. 2. Find the solid content of a wall whose lengt feet 6 inches, height 10 feet 9 inches, and thickness 2 f

	feet.	in.	
	48	6	
	10	9	
	485	0	
	36	4	6
	521	4	6
	2		
Content =	1042	9	= 1042 cubic feet, 9 parts

3. What is the value of a marble slab at 7*s*. per fo length being 6 feet 4 inches, and breadth 3 feet 6 inche

	feet.	in.
Length =	6	4
Breadth =	3	6
	19	0
	3	2
Area of slab =	22	2
		7
	2,0)155	2
Ans.	£7 15	2

4. A gentleman wishing to erect a marble chimney-p his dining-room, applied to a mason, who gave in the fo estimate:

	feet.	in.
The length of the mantel and slab, each	4	8
Sum of their breadths	3	4
Length of each jamb	4	6
Sum of their breadths	1	10

d the superficial content of the marble required to complete ꞉ job ; and determine the value thereof at 7*s*. per square foot.

	feet.	*in.*			*feet.*	*in.*
	4	8			4	6
	3	4			1	10
	14	0			4	6
	1	6	8		3	9
	15	6	8		8	3
	8	3				
›ntent req. =	23	9	8			
			7			
2,0)	166	7	8			
	£8	6	$7\frac{2}{3}d.$ = value required.			

SLATER'S AND TILER'S WORK.

Ex. 2. Find the expense of slating a barn at 30 shillings per ıare; the length being 43 feet 10 inches, the breadth on the : 27 feet 6 inches; the eave-boards projecting 15 inches on ꞉h side.

Now, it is customary to reckon the flat and half flat equal the girt over the roof; hence

27·5 feet = the flat.
13·75 feet = $\frac{1}{2}$ flat.
2·5 feet = the eaves.

43·75 = girt of the roof.
43·8333 = length of the roof

```
     2191665
    3068331
   1314999
  1753332
00)1917·706875
   19·17706875 squares in the roof.
            30 shillings per square.
  575·31206250
            12
   3·74475000
             4
   2·97900000
```

Ans. £28 15*s*. $3\frac{1}{2}d.$ the expense required.

Or thus:

feet.	*in.*		
27	6		= the flat.
13	9		= ½ flat.
2	6		= eaves.
43	9		= girt of roof.
43	10		= length.
1881	3		
36	5	6	
1917	8	6	= area of roo[f]
11506	3	0	
		5	
100)57531	3	0	
575	3½*d.*		

Or £28 15*s.* 3½*d.* a

3. Determine the cost of tiling a stable, whose len feet, and breadth on the flat 8 yards; the eaves being ed to project one foot on each side, and the roof pitch; reckoning at 25*s.* per square.

Flat = 24 feet.
½ Flat = 12 do.
Eaves = 2 do.

Girt of the roof = 38
Length = 30

100)1140

11·4 = no. of squares in th[e]
25 = price per square.

570
228

20)2850

£14 5*s.* = cost required.

PLASTERER'S WORK.

2. The dimensions of a certain partition are 141 feet 6 in circuit, and 11 feet 3 inches high; how many square does it contain?

	feet.	in.	
	141	6	
	11	3	
	1556	6	
	35	4	6
9)	1591	10	6
Ans.	176	7	10 6

That is, 176 sq. yards, 7 sq. ft. 120½ sq. inches.

A certain partition measures 234 feet 8 inches in circuit, 4 feet 6 inches in height, and is rendered between quarters. lathing and plastering cost 10*d.* per square yard, and the ring 4*d.* per yard; what will be the amount of the plas- bill?

	feet.	in.		
Circuit =	234	8		
Height =	14	6		
	3285	4		
	117	4		
	9)3402	8		sq. feet.
	378	1		= whole area of the partition nearly.
	75	5	6	= ⅕th thereof.
	302	6	7	= diff. plastered.
	453	5	7	= sum coloured.

	sq. yds.	*feet.*	*pts.*		*sq. yds.*	
Also	453	5	7	=	453·6203	
					4	= price per yard.
					12)1814·4812	
					2,0)15,1 $2\frac{1}{4}$	
					£7 11*s.* $2\frac{1}{2}$*d.*	

	sq. yds.	*feet.*	*pts.*			
Now	302	6	7	=	302·7314	sq. yards.
					10	= price per yard.
					12)3027·314	pence.
					2,0) 25,2 $3\frac{1}{4}$	
					12 12 $3\frac{1}{4}$	
					7 11 $2\frac{1}{2}$	
					£20 3 $5\frac{3}{4}$	= amount required.

4. The length of a room is 15 feet 6 inches, its breadth 13 feet 3 inches, and altitude 9 feet 6 inches, reckoning to the lower edge of the cornice, whose girt is $9\frac{1}{2}$ inches, and projection from the wall next to the ceiling is 5 inches. Now if in the room there be a door whose dimensions are 7 feet by 3½ feet, what will be the quantity of rendering and ceiling in the room; and what will be the cost thereof, supposing the cost of rendering to be 6*d.* per square yard, and the ceiling 1*s.* 3*d.* per square yard?

	feet.	*in.*		
Breadth of room =	13	3		
Breadth of cornice on both sides of the room.		10 inches.		
Breadth of ceiling =	12	5		
Length of ceiling =	14	8		
	173	10		
	8	3	4	
	182	1	4	= area of ceiling.

```
                 feet. in.
       Length  =  15   6
      Breadth  =  13   3
                 --------
          Sum  =  28   9
                       2
                 --------
Girt or circuit = 57   6
       Height  =   9   6
                 --------
                 517   6
                  28   9
                 --------
                 546   3
    7 × 3¾  =     26   3 = area of door.
                 --------
               9)520   0 = area of part rendered.
                 --------
                  57·777 sq. yards.
                       6
                 --------
                 346·662 pence.
                 £1 8s. 10½d. = cost of rendering

                    feet. in.
Length of cornice =  57   6
or breadth of do. =       9   6
                    -----------
                     43   1   6
                      2   4   9
                    -----------
                     45   6   3 = area of cornice.
                    182   1   4 = area of ceiling.
                    -----------
                    227   7   7
                 or 227·6319 sq. feet.
                 9)---------
                     25·2924 sq. yards.
          3d. is ¼    6·3231
                    ---------
                     31·6155 shillings.
                  or £1 11s. 7¼d. = cost of ce
```

In this question we have reckoned the cornice at the sa rate as the ceiling, but in general it is much more costly: value depending chiefly upon the quantity of labour besto in the making of it. Highly ornamented cornices are expensive.

OF VAULTED AND ARCHED ROOFS.

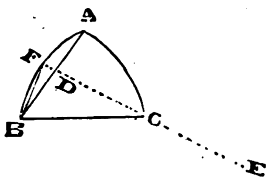
PROBLEM I.

To find the solid content of circular, elliptic, gothic, and cycl vaulted roofs.

Ex. 5. Determine the solidity of a semi-circular vault w span is 60 feet, and length 150 feet.

·7854	= area of circle to diam. 1.
3600	
4712400	
23562	
2)2827·4400	
1413·72	= area of the end of vault.
150	= length of the vault.
7068600	
141372	
Solidity = 212058·00	cubic feet.

6. What is the content of a gothic vault whose length feet, span 50 feet; the chord of either arc forming the being 86 feet, and the versed sine of either arc being 1



$$\text{ED} \times \text{DF} = \text{DB}^{2}*$$

$$\text{ED} \times 16 = 43^{2}$$

$$\text{ED} = \frac{43^{2}}{16} = \frac{1849}{16} = 115{\cdot}5625$$

$\therefore$ EF $= 131{\cdot}5625 =$ diameter of the gothic arc.

Also $\frac{16}{131{\cdot}5625} = {\cdot}121\,\frac{8}{13}$ nearly.

ıd tab. area corr. to $\cdot 121\,\frac{8}{13} = {\cdot}054438$

$17308{\cdot}6914 = (131{\cdot}5625)^{2}$ nearly.

```
  1384695312
  519260742
 692347656
692347656
865434570
```

ea of seg. AFBDA - $942{\cdot}2505424332$
2

ea of the two seg. $= 1884{\cdot}5010848664$
ea of $\triangle$ ABC $= 2057{\cdot}150$
ea of one end of the vault. $3941{\cdot}6510848664$
80 = length.

Content $= 31533{\cdot}20867893120$ cubic feet.

7. Find the content of a cycloidal vault whose span is 39 t, and length 100 feet.

Γhe nature of the cycloid is such that the span of the vault

Though in the fig. above, C *appears to be the centre of arc* AFB, *yet not necessarily so.*

is equal to the circumference of the generating circle; als area of the cycloid is equal to 3 generating circles; hence

$$
\begin{array}{r}
\cdot 07958 \\
1521 = 39^2 = (\text{span})^2 \\
\hline
7958 \\
15916 \\
39790 \\
7958 \\
\hline
121 \cdot 04118 \\
3 \\
\hline
363 \cdot 12354 = \text{area of cycloi} \\
100 = \text{length.} \\
\hline
\text{Content} = 36312 \cdot 354 \text{ cubic feet.}
\end{array}
$$

PROBLEM II.

To find the concave, or convex surface, of circular, elliptic, and cycloidal vaulted roofs.

Ex. 2. What is the concave surface of a cycloidal vaul' length is 55 feet, and height 14 feet?

The length of a cycloid = 4 times the axis or hei;

∴ length of the cycloid in this case = 56 feet.

Length of the vault = 55 feet.

$$
\begin{array}{r}
\hline
280 \\
280 \\
\hline
\text{Concave surface of vault} = 3080 \text{ sq. feet.}
\end{array}
$$

PROBLEM III.

To find the solid content of a dome; its height, and the dia of its base being known.

Ex. 2. In an octagonal spherical dome, one side of is 18 feet; what is the solidity?

Tab. multiplier = 4·828427 = area of oct. whose side is 1.
324 = 18^2

19313708
9656854
14485281

1564·410348 = area of the octagonal base.
10·242 = $\frac{2}{3}$ the altitude.*

3128820696
6257641392
3128820696
1564410348

Content = 16022·690784216 cubic feet.

PROBLEM IV.

To find the superficial content of a spherical dome.

Ex. 2. Determine the cost of painting an octagonal spherical dome, at 1*s*. 8*d*. per square yard; each side of the octagonal base being 12 feet.

4·828427 = area of oct. whose side is 1.
144 = 12^2

19313708
19313708
4828427

695·293488
2

9)1390·586976 sq. feet.
154·509664 sq. yards.
20*d*. = cost per yard.

3090·193280
= £12. 17*s*. 6*d*. = cost required.

* This altitude is thus determined: the side of the octagon is 18 feet, hence the radius of the circumscribing circle = 30·728, which expresses the altitude of the vault: and $\frac{1}{3}$ds of 30·728 = 10·242 = $\frac{2}{3}$ds the altitude of the vault. For the method of determining the radius of the circle from knowing a side of the inscribed octagon, see Problem XXII. in the Appendix to the Algebra.

PROBLEM V.

To find the solid content of a saloon.

Ex. 2. A circular building of 40 feet diameter, and
high to the ceiling, is covered with a saloon whose circul
is 5 feet radius; required the capacity of the room in cut

Since the building is 25 feet high to the ceiling, a
radius of the arch is 5 feet, hence from the nature of the
the altitude of the wall upon which the saloon rests is e
20 feet.

$$
\begin{array}{r l}
\cdot 7854 & = \text{area of circle to diam} \\
1600 & = 40^2 \\
\hline
4712400 & \\
7854 & \\
\hline
1256\cdot 6400 & \\
20 & = \text{altitude.} \\
\hline
25132\cdot 8000 & = \text{content of cyl} \\
& \text{[part of th}
\end{array}
$$

Again, since the arc on each side projects 5 feet, ar
the diameter of the building is 40 feet, therefore the d
of the ceiling = 30 feet: hence

$$
\begin{array}{r l}
3\cdot 1416 & \\
30 & \\
\hline
4)94\cdot 248 & = \text{perimeter of ceil} \\
\hline
23\cdot 562 & = \tfrac{1}{4}\ \text{perimeter.} \\
5 & = \text{height of arc.} \\
\hline
117\cdot 810 & \\
5 & = \text{projection.} \\
\hline
589\cdot 050 & \\
3\cdot 1416 & \\
\hline
353430 & \\
58905 & \\
235620 & \\
58905 & \\
176715 & \\
\hline
1850\cdot 559480 & = \text{A.}
\end{array}
$$

Next 40 = diameter of the room.
30 = diameter of the ceiling.

inder=10 the sq. of which = 100
·7854

78·54
10

3)785·40

261·8 = B.

78·54
900 = 30^2

of flat part of he ceiling } = 706·8600
5 = height of arch.

3534·3
1850·55948 = A.
261·8 = B.

5646·65948 = content of saloon.
25132·8 = content of cylindrical part of the room.

pacity of the room = 30779·45948 cubic feet.

PROBLEM VI.

ind the solid content of the vacuity formed by a groin arch, either circular or elliptical.

x. 2. What is the solid content of the vacuity formed by *lliptical groin*, one side of its square base being 20 feet *he height 6 feet?*

```
                 20
                 20
                ----
Area of base =  400
                  6 = height.
                ----
                2400
                ·904
               ------
              361600
             1808
              ------
Solidity = 2169·6 cubic feet.
```

PROBLEM VII.

To find the concave superficies of a circular gro

Ex. 2. What is the concave superficies of a circu arch, one side of its square being 9 feet?

```
             9
             9
            --
            81
        1·1416
        ------
         11416
        91328
        ------
Surface 92·4696 sq. feet.
```

3. Find the concave superficies of a circular gro ·· of its square being $14\frac{1}{4}$ feet.

```
        14·25
        14·25
       ------
         7125
        2850
       5700
      1425
     --------
     203·0625
       1·1416
     --------
     12183750
     2030625
    8122500
   2030625
  2030625
  ----------
```

Content = 231·81615 square feet.

OF TIMBER MEASURE.

PROBLEM I.

find the area, or superficial content, of a board or plank.

. 2. What is the content of a board, whose length is inches, and breadth 1 foot 10 inches?

```
          feet.  in.
           5     7
           1    10
          ---------
           5     7
           4     7   10
          ------------
```

Superficial content = 10 2 10 as required

3. At $1\frac{1}{2}$d. per foot, what is the value of a plank whose length is 12 feet 6 inches, and breadth 11 inches throughout?

	feet.	in.	
	12	6	
		11	
	11	5	6
$\frac{1}{2}$d. is $\frac{1}{2}$	5	8	9
	17	2	3 = 1*s*. $5\frac{1}{6}$*d*. nearly, the value required.

4. Find the value of 5 oaken planks at 3*d*. per foot, each being $17\frac{1}{2}$ feet long, and their particular breadths as follows, viz: two $13\frac{1}{2}$ inches, and one of $14\frac{1}{2}$ inches in breadth; also the two remaining ones, each 18 inches at the broader end and $11\frac{1}{4}$ at the narrower.

in.
13·5
13·5
14·5
14·625 } mean breadth of the two
14·625 } last named planks.

Sum of breadths = 70·75 inches.
12)―――――
5·89583 &c. feet.
17·5 = common length.

2947915
4127081
589583

103·177025 square feet.
3

12)309·531075

25 $9\frac{1}{2}$ = £1 5s. $9\frac{1}{2}$d. the value.

PROBLEM II.

To find the solidity of squared or four-sided timber.

x. 2. The length of a piece of timber is 24·5 feet, and its s are equal squares, whose sides are each 1·04 feet: what is solidity?

```
     1·04
     1·04
     ----
     416
    104
    ------
    1·0816
      24·5
    ------
     54080
    43264
   21632
  --------
  26·49920
        12
  --------
   5·99040
```

ns. = 26 cubic feet, 6 parts nearly; each of which parts $\frac{1}{12}$ of a cubic foot.

The length of a piece of timber is 20·38 feet; and the are unequal squares; a side of the greater square being inches, and a side of the lesser $9\frac{7}{8}$ inches; what is the ity?

$\frac{1}{8}$ = 19·125
$\frac{7}{8}$ = 9·875

2)29

14·5 = *1·20833* feet = mean breadth and thickness.

```
        1·20833
        1·20833
       ---------
         362499
        362499
       966664
     241666
    120833
   ------------
   1·4600613889
          20·38
   ------------
   116804811112
   43801841667
 29201227778
 --------------
```

Content = 29·756051005782 cubic feet.

4. The length of a piece of timber is 27·36 feet; greater end the breadth is 1·78 feet, and the thickn feet; and at the lesser end the breadth is 1·04 feet, thickness ·91 feet: what is the solidity?

```
Greater breadth = 1·78     Greater thickness =
Lesser breadth  = 1·04     Lesser thickness  =
                  ----
               2)2·82                        2)
                  ----
  Mean breadth = 1·41       Mean thickness =
                 1·07
                 ----
                  987
                141
                ------
                1·5087
                 27·36
                ------
                 90522
                45261
              105609
              30174
              ---------
Content =     41·278032 cubic feet.
```

PROBLEM III.

To find the solidity of round or unsquared timber.

RULE I.

. 2. The length of a tree is 25 feet, and the girt through-
½ feet : what is its solidity?

$$
\begin{array}{rr}
\text{Girt} = & 2{\cdot}5 \\
\tfrac{1}{4}\text{ girt} = & {\cdot}625 \\
 & {\cdot}625 \\
\hline
 & 3125 \\
 & 1250 \\
 & 3750 \\
\hline
\text{Sq. of } \tfrac{1}{4}\text{ girt} = & {\cdot}390625 \\
 & 25 = \text{length.} \\
\hline
 & 1953125 \\
 & 781250 \\
\hline
 & 9{\cdot}765625 \\
 & 12 \\
\hline
 & 9{\cdot}187500 \\
\hline
\end{array}
$$

Ans. 9 cubic feet, 9 parts.

The length of a tree is 14½ feet, and its girt in the mid-
·15 feet : required the solidity.

$$
\begin{array}{rr}
\text{Girt} = & 3{\cdot}15 \\
\tfrac{1}{4}\text{ girt} = & {\cdot}7875 \\
 & {\cdot}7875 \\
\hline
 & 39375 \\
 & 55125 \\
 & 63000 \\
 & 55125 \\
\hline
\text{quare of } \tfrac{1}{4}\text{ girt} = & {\cdot}62015625 \\
 & 14{\cdot}5 = \text{length.} \\
\hline
 & 310078125 \\
 & 248062500 \\
 & 62015625 \\
\hline
\end{array}
$$

Content = $\underline{8{\cdot}992265625}$ = 9 cubic feet nearly.

4. The circumference of a tree in 4 different place follows: in the first place 5 feet 9 inches, in the second 5 inches, in the third 4 feet 9 inches, and in the fourth 9 inches; and the length of the whole tree is 15 feet: the solidity?

	feet.	in.
	5	9
	4	5
	4	9
	3	9
Sum of girts =	18	8
4)		
Mean girt =	4	8
4)		
	1	2

	feet.	in.	
	1	2	
	1	2	
	1	2	
		2	4
	1	4	4 = sq. of
			15
Content =	20	5	0

Ans. 20 cubic feet, 5

5. An oak tree is 45 feet 7 inches long, and its qua 3 feet 8 inches: what is the solid content, allowing $\frac{1}{12}$ bark?

	feet.	in.	
$\frac{1}{4}$ girt =	3	8	
		4	
Whole girt =	14	8	
Allow. for bark =	1	2	8
	13	5	4
4)			
Quarter girt =	3	4	4 = 3·36111 &c. feet.

```
                  3·36111
                  3·36111
                  -------
                  336111
                 336111
                336111
              2116666
             1008333
            1008333
            ------------
Square of ¼ girt = 11·3070604321
                         45·58 = length
            -------------
            904564834568
           565353021605
          565353021605
         452282417284
         ----------------
Content = 515·3758144495118 cubic fee
```

RULE II.

2. If the length of a tree be 14 feet 6 inches, and its irt 3·15 feet; what is its solidity?

$$
\begin{array}{rl}
\text{Mean girt} = & 3.15 \\
5) & \overline{} \\
& 0{\cdot}63 \\
& {\cdot}63 \\
\hline
& 189 \\
& 378 \\
\hline
& {\cdot}3969 = (\tfrac{1}{5}\text{ girt})^2 \\
& 29 = \text{twice the length.} \\
\hline
& 35721 \\
& 7938 \\
\hline
\text{Content} = & 11{\cdot}5101 \text{ cubic feet.}
\end{array}
$$

f the length of a tree be 24 feet, and the girt through- 'eet; what is the content?

$$
\begin{array}{rl}
& 5)8{\cdot} \\
\hline
& 1{\cdot}6 \\
& 1{\cdot}6 \\
\hline
& 96 \\
& 16 \\
\hline
& 2{\cdot}56 \\
& 48 = \text{twice the length.} \\
\hline
& 2048 \\
& 1024 \\
\hline
\text{Solidity} = & 122{\cdot}88 \text{ cubic feet.}
\end{array}
$$

If a tree girt 12 feet at the thicker end, and 3 feet at the *r end; required* the solidity when the length is 18 feet

```
            12
             3
         ―――
         2)15
         ―――
         5) 7·5 = mean girt.
         ―――
            1·5 = 1/5 mean girt.
         ―――
           2·25 = (1/5 mean girt)²
            36  = twice length.
         ―――
          1350
          675
         ―――
          81·00 cubic feet requir[ed].
```

5. A tree girts in five different places as follows: in the fi[rst] place 9·43 feet, in the second 7·92 feet, in the third 6·15 fe[et,] in the fourth 4·74 feet, and in the fifth 3·16 feet; and th[e] whole length is $17\frac{1}{4}$ feet; what is the solidity?

```
                  9·43
                  7·92
                  6·15
                  4·74
                  3·16
                5)31·40
Mean girt     = 6·28
                5)
                  1·256
                  1·256
                  7536
                 6280
                2512
               1256
               1·577536
                   34·5 = twice the length.
               7887680
              6310144
             4732608
Content = 54·4249920 cubic feet.
```

Greenwich park a workman took the girt of an oak in
nt places as follows: in the first place 10·5 feet, in the
} feet, in the third 6½ feet, in the fourth 3¼ feet, in the
eet, and in the sixth place 18 inches; determine the
of cubic feet in the tree, its length being 25 feet, and
stripped off.

```
          10·5
           8·
           6·5
           3·25
           2·
           1·5
        ------
        6)31·75
          -------
Mean girt =  5·2916
          5)-------
             1·0583 = 1/5 of mean girt.
             1·0583
            -------
              31749
             84664
            52915
           10583
          ----------
          1·11999889
                  50 = twice length.
          -------------
          55·99994450 = 56 cubic feet, nearly.
          -------------
```

SPECIFIC GRAVITY.

PROBLEM I.

ng given the weight of a body, to find its magnitude.

How many cubic inches are there in one pound avoir-
ight of gunpowder?

As 992 : 16 : : 1728 : content requi
16

10368
1728

922)27648

30 cubic inches.

3. How many cubic feet are there in a ton of specific gravity is 925 ?

oz. in 1 *ton. cub. foot.*
As 925 : 35840 : : 1 : answer requir
1

925)35840(38$\frac{115}{184}$ cubic feet, the
2775

8090
7400

690

4. How many cubic inches are there in 10 p pure silver ?

Now 1 lb. avoir. : 1 lb. troy : : 17 : 14 n
∴ 1 lb. troy = $\frac{14}{17}$ of 1 lb. avoir.
And 10 lbs. troy = $\frac{140}{17}$ lbs. avoir. = $\frac{140}{17}$ ×
= 131·76 oz. avoir.

sp. grav. oz. avoir. cub. in.
Hence 10474 : 131·76 : : 1728 : answer
1728

10474)227681·28

Ans. 21·73 cubic inches.

* See Appendix to the Arithmetic.

ound avoirdupois : a pound troy : : 13999 : 11520, which
rer to the truth than that of 17 : 14; therefore if the
ratio be used, the answer to this question will be obtained
correctly than it has been done above. Thus:—

nd troy $= \frac{11520}{13999}$ lbs. avoir.

bs. troy $= \frac{115200}{13699}$ lbs. avoir.

$= \frac{115200 \times 16}{13999}$ avoir. oz. $=$ 131·6665 oz. avoir.

	sp. grav.		*oz. avoir.*		*cub. in.*
∴	10474	:	131·6665	: :	1728

```
                 131·6665
                     1728
                 --------
                 10533320
                 2633330
                9216655
               1316665
               ----------
10474)227519·7120(21·722, &c. cubic inches.
      20948                The answer required.
      -----
       18039
       10474
       -----
        75657
        73318
        -----
         23391
         20948
         -----
          24432
          20948
          -----
           3484
```

K

PROBLEM II.

Having given the magnitude of a body, to determine its w

Ex. 2. Find the weight of a block of marble, whose is 63 feet; its breadth and thickness being each 12 feet

$$\begin{array}{rr}
\text{Length} = & 63 \\
 & 12 \\
\hline
 & 756 \\
 & 12 \\
\hline
\text{Content} = & 9072 \text{ cubic feet.}
\end{array}$$

cub. ft. cub. ft. sp. grav.

Hence as 1 : 9072 :: 2700 : weight.

$$\begin{array}{rl}
 & 2700 \\
\hline
 & 6350400 \\
 & 18144 \\
\hline
16\left\{\begin{array}{l} 4)24494400 \\ \hline 4)6123600 \end{array}\right. & \text{avoir. ounces.} \\
\hline
28\left\{\begin{array}{l} 4)1530900 \\ \hline 7)382725 \end{array}\right. & \text{lbs.} \\
\hline
 & 80)54675 \\
\hline
\end{array}$$

Weight required $= 683\frac{35}{80}$ tons.

3. What is the weight of a pint of gunpowde asure?

cub. inches. cub. inches.

As 1758 : 34·659 (in one pint) : : 922

```
          922
       ------
        69318
       69318
      311931
     -------
1728)31955·598(18·492 ounces, the weight re-
     1728                              [quired.
     ----
     14675
     13824
     -----
       8515
       6912
       ----
       16039
       15552
       -----
         4878
         3456
         ----
         1422
```

What is the weight of a block of dry oak which measures ; in length, 3 feet in breadth, and 2½ feet deep?

```
    2·5
      3
    ---
    7·5
     10
    ---
Content = 75·0 cubic feet.
```

```
                                  sp. grav.
as 1 cub. foot : 75·0 cub. feet : : 925
                                     75
                                   ----
                                   4625
                                  6475
                                 -----
                              16)69375
```

Ans. 4335$\frac{15}{16}$ lbs. or 4336 lb [nea

PROBLEM III.

To find the specific gravity of a body.

CASE I.—*When the body is heavier than water.*

Ex. 2. A piece of alabaster weighs 108 pounds in air, a 69 pounds in water; determine the specific gravity of t alabaster.

$$\begin{array}{r} 108 \\ 69 \\ \hline \text{Weight lost} = 39 \end{array}$$

Hence as 39 : 108 : : 1000
1000

$$\begin{array}{l} 39)108000(2769 = \text{specific gravity required.} \\ \quad 78 \\ \quad \overline{300} \\ \quad 273 \\ \quad \overline{\ 270} \\ \quad\ 234 \\ \quad \overline{\ \ 360} \\ \quad\ \ 351 \\ \quad \overline{\qquad 9} \end{array}$$

3. A piece of stone weighs 10 pounds in air, and 6 pou 12 ounces in water; determine the specific gravity of the sto

$$\begin{array}{r} 10\ lbs. \\ 6\ 12 \\ \hline \text{Weight lost} = 3\ \ 4 \end{array}$$

oz. *oz.*
As 52 : 160 : : 100
160

52)160000(3077 nearly = specific gravity of
156 [the stone.

400
364

360

CASE II.—*When the body is lighter than water.*

Ex. 2. A piece of fir weighs 10 lbs. in air, and a piece of ı, which weighs 12 lbs. in air and 10 lbs. in water, is connected th it, and together they weigh 2 lbs. in water ; find e specific avity of the fir.

Tin weighs 12 lbs. in air.
10 lbs. in water.

1st rem. 2 = loss of weight.
ompound mass weighs 22 lbs. in air.
2 lbs. in water.

2d rem. 20
Hence as 18 : 10 : : 1000 : specific gravity of the fir.
10

18)10000(555 nearly = sp. grav. of fir.
90

100
90

100
90

10

PROBLEM IV.

To determine the quantities of two ingredients in a compound whose weight is given.

Ex. 2. A composition weighing 20 pounds, contains silver whose specific gravity is 10474, and fine copper, whose specific gravity is 8878; determine the weight of each ingredient: the specific gravity of the compound being 9000.

Sp. grav. of silver	= 10474		8878
Sp. grav. of compound	= 9000		9000
Difference	= 1474	Difference	= 122
	8878		10474
	4208172		1277828

10474
8878 = sp. grav. of fine copper.

1596 = difference.
9000

14364000

Then as 14364000 : 20 lbs. : : 4108172
20

14364000)84163440(5·859 lbs of silver,
71820000 and therefore 14·141 lbs. of copper.
123434400
114912000
85224000
71820000
134040000
129276000
4764000

ON THE WEIGHT AND DIMENSIONS OF BALLS AND SHELLS.

PROBLEM I.

Having given the diameter of an iron shot, to find its weight.

Ex. 2. The diameter of an iron shot is 4 inches; what is its weight?

$$\begin{array}{r} 64 = 4^3 = (\text{diameter})^3 \\ 9 \\ \hline 64)576 \\ \hline \textit{Ans.}\ \ 9 \text{ lbs.} \end{array}$$

3. The diameter of an iron shot is 6·5 inches; what is its weight?

$$\begin{array}{r} 6{\cdot}5 \\ 6{\cdot}5 \\ \hline 325 \\ 390 \\ \hline 42{\cdot}25 \\ 6{\cdot}5 \\ \hline 21125 \\ 25350 \\ \hline 274{\cdot}625 \\ 9 \end{array} = (\text{diameter})^3$$

$$\begin{array}{l} 64)\overline{2471{\cdot}625}(38{\cdot}619 \text{ lbs.} = \text{weight required.} \\ \quad 192 \\ \quad \overline{551} \\ \quad 512 \\ \quad \overline{396} \\ \quad 384 \\ \quad \overline{122} \\ \quad 64 \\ \quad \overline{585} \\ \quad 576 \\ \quad \overline{9} \end{array}$$

PROBLEM II.

Having given the diameter of a leaden ball, to find its wei

Ex. 2. Determine the weight of a leaden ball whose dia is 4·5 inches.

$$\begin{array}{r} 4{\cdot}5 \\ 4{\cdot}5 \\ \hline 225 \\ 180 \\ \hline 20{\cdot}25 \\ 4{\cdot}5 \\ \hline 10125 \\ 8100 \\ \hline 91{\cdot}125 = (\text{diameter})^3 \\ 2 \\ \hline 9)182.25 \\ \hline \textit{Ans.}\ 20{\cdot}25\ \text{lbs.} \end{array}$$

3. What is the weight of a leaden ball, whose circumfe or girt is 16 inches?

$$\begin{array}{l} 3{\cdot}1416)16{\cdot}0000(5{\cdot}093 \text{ nearly} = \text{diameter.} \\ \qquad\quad 157080 \\ \hline \qquad\qquad 292000 \\ \qquad\qquad 282744 \\ \hline \qquad\qquad\quad 92560 \end{array}$$

Also $(5\cdot093)^3 = 132\cdot105$ nearly

$$\begin{array}{r} 2 \\ \hline 9)264\cdot210 \\ \hline \end{array}$$

Weight = 29·356 lbs.: the answer.

PROBLEM III.

ving given the weight of an iron ball, to find its diameter.

:. 2. The weight of an iron ball is 45 lbs.; determine its ieter.

$$\begin{array}{rl} \text{Weight} = & 45 \text{ lbs.} \\ & 64 \\ \hline & 180 \\ & 270 \\ \hline 9) & 2880 \\ \hline \end{array}$$

320 whose logarithm is 2·5051500
3)————
Diameter = 6·84 the no. corr. to ·8350500

The weight of an iron ball is 18 lbs.; determine its dia-
:r.

$$\begin{array}{rl} \text{Weight} = & 18 \text{ lbs.} \\ & 64 \\ \hline & 72 \\ & 108 \\ \hline 9) & 1152 \\ \hline \end{array}$$

128 whose logarithm is 2·1072100
3)————
Diameter = 5·(39 the no. corr. to ·7024033

PROBLEM IV.

• *Having given the weight of a leaden ball, to find its diam*

Ex. 2. Find the diameter of a leaden ball, whose wei
45 lbs.

```
Weight — 45 lbs.
          289
         ----
         1445
        1156
        -----
        13005 whose log. is 4·1131104
                             3)-------
    4)23·498 the no. corr. to 1·3710368
      ------                 ---------
Diameter = 5·874 inches.
```

3. The weight of a leaden ball is 12 lbs.; find its dia

```
Weight = 12 lbs.
          289
         ----
         3468 whose log. is 3·5400791
                             3)-------
    4)13·122 the no. corr. to 1·1800263
      ------                 ---------
Diameter = 3·28 inches.
```

PROBLEM V.

Having given the external and internal diameters of a shell, to find its weight.

Ex. 2. Find the weight of a 21 inch iron bomb-she
·an thickness is 3 inches.

$21^3 = 9261$
$15^3 = 3375$

5886
9

64)52974(827·7 lbs. the weight required.
512
177
128
494
448
460

What is the weight of a 9 inch iron bomb-shell, whose thickness is $1\frac{1}{2}$ inches?

$9^3 = 729$
$6^3 = 216$

513
9

64)4617(72·14 lbs. — weight required.
448
137
128
90
64
260
256
4

PROBLEM VI.

d the number of pounds of powder which can be contained in a hollow shell whose internal diameter is given.

2. *How* many pounds of powder are required to fill a *shell,* whose internal diameter is 13 inches?

```
         13
         13
        ---
        169
         13
        ---
        507
       169
      -----
59·32)2197·0(37·03 lbs. the answ
      17796
      -----
      41740
      41524
      -----
       21600
       17796
       -----
        3804
```

3. Required the same as in the last Problem, for a shell internal diameter is 15 inches.

```
         15
         15
        ---
        225
         15
       ----
       1125
       225
      -----
59·32)3375·00(56·89 lbs. the answer
      29660
      -----
       40900
       35592
       -----
        53080
        47456
        -----
         56240
         53388
         -----
          2852
```

PROBLEM VII.

find the dimensions of a cubical box to hold a given quantity of powder.

x. 2. Determine the length of a cubical box which will hold bs. of powder..

```
         31·06
            14
      --------
        434·84 whose log. is 2·6383295
                                  3)---------
Length = 7·576 inches ·8794431
```

PROBLEM VIII.

ing given the length, breadth, and depth of a rectangular box, to find how many pounds of powder will fill it.

x. 2. The length of a rectangular box is 2 feet 3 inches, dth 1 foot 9 inches, and depth 15 inches; how many pounds owder will the box contain?

```
Length   = 27
Breadth  = 21
          ----
            27
           54
          ----
           567
Depth    = 15
          ----
          2835
          567
          ----
          8505
         ·0322
         -----
         17010
        17010
       25515
       -------
Ans. 273·8610 lbs.
```

PROBLEM IX.

Having given the diameter and length of a hollow cylind determine how many pounds of gunpowder will fill i

Ex. 2. The diameter of a hollow cylinder is 8 inches length 16 inches: how many pounds of powder will it c

$$
\begin{array}{r l}
8 & \\
8 & \\
\hline
64 & = \text{diameter}^2. \\
16 & = \text{length.} \\
\hline
384 & \\
64 & \\
\hline
4{,}0)1024 & \\
\hline
\textit{Ans.}\ 25{\cdot}6 & \text{lbs.} \\
\hline
\end{array}
$$

3. The diameter of a hollow cylinder is 2 feet, and its l 3 feet; how many pounds of powder will it contain?

Diameter 24 inches.

$$
\begin{array}{r l}
24 & \\
\hline
96 & \\
48 & \\
\hline
576 & = (\text{diameter})^2 \\
36 & = \text{length.} \\
\hline
3456 & \\
1728 & \\
\hline
4{,}0)20736 & \\
\hline
\textit{Ans.}\ 518{\cdot}4 & \\
\hline
\end{array}
$$

PROBLEM X.

Having given the diameter of a hollow cylinder, to find what portion of the cylinder will be occupied by a given quantity of powder.

Ex. 2. The diameter of a hollow cylinder is 18 inches; now if 25 lbs. of powder be poured in, how much of the cylinder will be occupied by the powder?

Weight = 25 lbs.
40

324)1000(3.086 inches, the altitude at which the powder will stand in the cylinder.
972

2800
2592

2080
1944

136

3. The diameter of a hollow cylinder is 6 inches. If, then, 12 lbs. of powder be poured into the cylinder, what portion of the cylinder's axis will be covered by the powder?

Weight = 12 lbs.
40

36)480(13.33 the portion of the cylinder's axis occupied by the powder.
36

120
108

120
108

12

4. If the diameter of a hollow cylinder be 10 inches; portion of the cylinder's axis will be covered, if 9 lbs. of po be poured into the cylinder?

Weight = 9 lbs.
40

100)360

Ans. 3·6 inches = that portion of the cylir axis, occupied or covered by the powder.

THE PILING OF BALLS AND SHELLS

PROBLEM I.

To find the number of shot in a finished triangular pile

Ex. 2. Determine the number of shot in a complet finished triangular pile, one side of whose base contai balls.

15 = No. in a side.
16 = the No. + 1.

90
15

240
17 = the No. + 2.

1680
24

6)4080

Ans. 680 shot.

[I]f a side of the base of a triangular pile contain 18 shot, [how m]any are there in the pile?

18 = the No.
19 = the No. + 1.

162
18

342
20 = the No. + 2.

6)6840

Ans. 1140 shot.

[F]ind the number of shot in a complete triangular pile, [a sid]e of whose base contains 26 shot.

26 = the No.
27 = the No. + 1.

182
52

702
28 = the No. + 2.

5616
1404

6)19656

Ans. 3276 shot.

PROBLEM II.

[T]o find the number of shot in a finished square pile.

2. *Find* the number of shot in a finished square pile, [a side] *of whose base* contains 21 shot.

21 = the No.
22 = the No. + 1.

42
42

462
43 = 1 + double the No.

1386
1848

6)19866

Ans. 3311 shot.

3. Determine the same as in the last problem, whe
of the base contains 41 shot.

41 = the No.
42 = the No. + 1.

82
164

1722
83 = 1 + twice the No.

5166
13776

6)142926

Ans. 23821 shot.

PROBLEM III.

find the number of shot in a finished rectangular pile.

2. How many shot are there in a finished rectangular
ie length of whose base contains 40 shot, and breadth
ns 22 shot?

$$
\begin{array}{r l}
40 & = \text{No. of shot in the base.} \\
3 & \\
\hline
120 & \\
1 & \\
\hline
121 & \\
22 & = \text{No. in the breadth.} \\
\hline
99 & \\
23 & \\
\hline
297 & \\
198 & \\
\hline
2277 & \\
22 & \\
\hline
4554 & \\
4554 & \\
\hline
6)50094 & \\
\hline
\end{array}
$$

Ans. 8349 shot.

PROBLEM IV.

*determine the number of shot contained in a pile which is
iished, provided the highest course of balls be complete, and
imber contained in each side thereof be given.*

. 2. How many shot are contained in an unfinished
rular pile, a side of whose base contains 25 shot, and a
f the uppermost course 14 shot?

$$\begin{array}{r} 25 \\ 26 \\ \hline 150 \\ 50 \\ \hline 650 \\ 27 \\ \hline 4550 \\ 1300 \\ \hline 6)17550 \\ \hline 2925 \end{array}$$

2925 = No. of shot, if the pile were complet

$$\begin{array}{r} 13 \\ 14 \\ \hline 52 \\ 13 \\ \hline 182 \\ 15 \\ \hline 910 \\ 182 \\ \hline 6)2730 \\ \hline 455 \end{array}$$

455 = No. of shot in a little pile, which being upon the unfinished pile will make it complete.

Hence 2925
455

Ans. = 2470 shot.

3. How many balls are contained in an incomplete pile, a side of whose base contains 20 shot, and a sid uppermost course 8 shot?

```
      20
      21
     ———
     420
      41
     ———
     420
    168
   ———
 6)17220
   ————
    2870 = No. in the pile, if complete.
   ————
       7
       8
     ———
      56
      15
     ———
     280
     56
    ————
  6)840
   ———
    140 = No. in the little pile.
```

Hence 2870
140

Ans. = 2730 shot.

Determine the number of shot contained in an unfinished ıngular pile, of 15 courses, the No. of shot in the length breadth of the base being 45 and 40.

ow since there are 15 courses, and the number of shot in *side decreases by 1* for each course; therefore the length *›readth of the* highest course are 31, 26. Hence we must *he number of* shot in a rectangular pile, the length an *h of whose* base contain 30 and 25 shot respectively.

45
3
135
1
136
40
96 = 3 $m - n + 1$
41
96
384
3936
40
6)157440
26240 = No. of shot in the pile, if

30
3
90
1
91
25
66
26
396
132
1716
25
8580
3432
6)42900
7150 = No. of balls in the little rectang
necessary to complete the pile mentioned in the questi

26240
7150
Ans. = 19090 shot.

* In every pile there may be found three parallel rows of balls,
which being multiplied by the number of balls in the triangul
then divided by 3, will give the whole number of balls in the pil

For $\frac{n}{6}\cdot(n+1)\cdot(3m-n+1) = \frac{n.(n+1)}{2}\cdot\frac{(2m+}{3}$

EXAMPLES
IN THE DETERMINATION OF DISTANCES
BY THE VELOCITY OF SOUND.

x. 2. Being at sea off Fort William, I saw the flash of a non, and counted 8 seconds by my watch, between the h and report; find my distance from Fort William.

```
              feet.
Sound moves 1142 in 1″
               8
            ------
          3)9136
            -------
      1760)3045·33(1·72 miles, the answer req.
           1760
           ------
           12853
           12320
           ------
             5333
             3520
             ----
             1813
             ----
```

3. Having observed a flash of lightning proceed from a rk cloud, I counted 4 seconds, and then heard the clap of under: how far was I from the cloud?

But $\frac{n.(n+1)}{2}$ = the No. of balls in the triangular end or face of the pile; r the face is formed by balls commencing with 1 at the top, and increasing units to the number of balls in the breadth of the base. And $2m + \epsilon$ = uble the number of balls in the length of the base + the single row of balls the top. Hence, double the number of balls in the length of the base increased by the number of balls in the top row, being multiplied by the number balls in the triangular face, and the product divided by 3 will give the umber of balls in the pile. In the square pile, the sum of twice the number f balls in one side, together with the single ball at the top, is to be multiplied y the number of balls in a triangular face; and in the triangular pile, the umber of balls in a side increased by one, and added to the single ball e top, must be multiplied by the number of balls in a triangular face, ch of the products divided by 3 will give the number of balls in the resp e pile.

1142
4
___
3)4568 feet.
___
1760)15226(·8651 of a mile; or $\frac{9}{10}$ of a mile nearl
14080
___
11460
10560
___
9000
8800
___
2000
1760
___
240
___

4. A woodman hewing timber struck so vigorousl heard each stroke of his axe, and counted 5 beats of n between each stroke; how far was the woodman dista me, reckoning 6 pulsations to elapse during the time th takes in moving one mile?

As 6 pulsations : 1760 yards : : 5 pulsations :
5
___
6)8800
___
Ans. = $1466\frac{2}{3}$ yards.

CASK GAUGING.

ALL the examples which are given in the Mens there solved; consequently it may appear unnecess

e the subject of gauging into this key; but as the principles he science are not given, I have thought proper to give, in volume, a short account of the subject. Gauging is one of branches of mensuration, and teaches us how to find the tents of vessels in gallons, or in some measure of capacity. measures are always taken in inches, or at least must be uced to inches, and the contents are always computed in ons; for which purpose, certain *fixed numbers* are made of to facilitate the computation. These *fixed numbers* l be explained in due time. In the gauging of malt, however, content is estimated in bushels.

In finding the *areas* of all plane surfaces in gauging, their tents are expressed in gallons, because in the business of ging, every area or surface is supposed to be one inch deep; l, therefore, in the language of a gauger, the area of a cle, a square, &c. is a certain number of gallons. For there the same number of superficial inches in any plane figure, there are solid inches of one inch deep; so that, what is led a surface in gauging, is in reality a solid, whose height is e inch.

The sliding rules of Leadbetter and Verie are fully described the mensuration, I shall therefore say nothing in this place pecting them.

The *fixed numbers* above mentioned are certain *multipliers, isors,* and *gauge points.*

find proper multipliers, divisors, and guage points for square figures.

Since 277·274 cubic inches make one imperial gallon, and 18·192 cubic inches make one imperial bushel, therefore the ntent of any vessel in cubic inches being divided by 277·274 ll give the content in imperial gallons; and if divided by 18·192 will give the content in imperial bushels.

Instead of these divisors, multipliers may be found which ll answer the same purpose; for $\frac{1}{277 \cdot 274} = \cdot 003607$; and $\frac{1}{18 \cdot 192} = \cdot 000451$; hence the content of any figure or sel in cubic inches being multiplied by ·003607 will give th luct in imperial gallons; and the same content in inch iplied by ·000451 will give the product in imperial bus

The *gauge points* for square figures are the square roots of 727,·274 and 2218·192; and are, therefore, the sides of two squares, one of whose areas is a gallon, and the other a bushel.

$\sqrt{277 \cdot 274} = 16 \cdot 65$ the square gauge point for gallons.

$\sqrt{2218 \cdot 192} = 47 \cdot 10$ the square gauge point for bushels.

The second gauge points for squares are found by multiplying or dividing the former gauge points by $\sqrt{10}$ or by 3·162; hence

$\sqrt{2772 \cdot 74} = 52 \cdot 64 =$ second square gauge point for gallons.

$\sqrt{221 \cdot 8192} = 14 \cdot 98 =$ second square gauge point for bushels.

All *guage points* are the first terms of any proportion upon the lines D and C; and these second gauge points serve, when the terms run off the lines. For the numbers on D being as the square roots of those on C; the variation of 10 on C will cause a variation of the gauge point on *D*, equal to the square root of 10.

To find proper multipliers, divisors, and gauge points, for Circles.

Let $\Delta =$ the diameter of any circle in inches, then $\cdot 7854 \times \Delta^2 =$ the area of that circle in inches; and $\frac{\cdot 7854}{277 \cdot 274} \Delta^2 =$ the area or content in imperial gallons; and $\frac{\cdot 7854}{2218 \cdot 192} \Delta^2 =$ the area or content in imperial bushels.

Now $\frac{\cdot 7854}{277 \cdot 274} = \cdot 002833$; and $\frac{\cdot 7854}{2218 \cdot 192} = \cdot 000356$;

Hence we obtain

$\cdot 002833 \times \Delta^2 =$ the content of a circle whose diameter is Δ, in imperial gallons.

And $\cdot 000356 \times \Delta^2 =$ the content of a circle whose diameter is Δ, in imperial bushels.

Therefore ·002833, and ·000356 are the proper multipliers for gallons and bushels.

The proper divisors are formed thus: since multiplying by $\frac{\cdot 7854}{277 \cdot 274}$ is equivalent to dividing by $\frac{277 \cdot 274}{\cdot 7854}$, and multiplying

$\frac{7854}{18\cdot192}$ is equivalent to dividing by $\frac{2218\cdot192}{\cdot7854}$;

Therefore $\frac{\Delta^2}{353\cdot04}$ will give the quotient in imperial gallons;

And $\frac{\Delta^2}{2824\cdot29}$ will give the quotient in imperial bushels.

The *gauge points* for circles are the diameters of such circles contain some known measure, as a gallon. Thus.

$$\cdot7854\ \delta^2 = 277\cdot274\ \therefore \delta = \left(\frac{277\cdot274}{\cdot7854}\right)^{\frac{1}{2}} = \sqrt{353\cdot04} = 18\cdot79$$

$$\text{d}\ \cdot7854\ \Delta^2 = 2218\cdot192\ \therefore \Delta = \left(\frac{2218\cdot192}{\cdot7854}\right)^{\frac{1}{2}} = \sqrt{2824\cdot29}$$

$= 53\cdot14$.

Therefore 18·79, 53·14 are the gauge points for a circle, in imperial gallons and bushels respectively.

The second gauge points are found by multiplying or dividing the former gauge points by $\sqrt{10}$, or by 3·162. Thus:

$\sqrt{3530\cdot4} = 59\cdot41$ the 2d gauge point for gallons.

$\sqrt{282\cdot429} = 16\cdot8$ the 2d gauge point for bushels.

To find proper multipliers, divisors, and gauge points for Spheres.

If Δ = the diameter of a sphere in inches; then $\cdot5236 \times$ = the content of that sphere in cubic inches. Also $\frac{\cdot5236}{277\cdot274}$ = the content in imperial gallons; and $\frac{\cdot5236}{2218\cdot192}\ \Delta^3$ = the content in imperial bushels.

But $\frac{\cdot5236}{277\cdot274} = \cdot0018884$, and $\frac{\cdot5236}{2218\cdot192} = \cdot000236$;

Hence $\cdot0018884 \times \Delta^3$ = the content of the sphere in imperial gallons.

And $\cdot000236 \times \Delta^3$ = the content of the sphere in imperial *bushels*.

Now *instead of* multipliers, if we wish for divisors, we ca[n] *them as follows.* The content of the sphere in gallons

$\frac{\cdot 5236}{277\cdot 274}\,\Delta^3 = \Delta^3 \div \frac{277\cdot 274}{\cdot 5236} = \frac{\Delta^3}{529\cdot 554}$; also the content in bushels $= \frac{\cdot 5236}{2218\cdot 192}\,\Delta^3 = \Delta^3 \div \frac{2218\cdot 192}{\cdot 5236} = \frac{\Delta^3}{4236\cdot 43}$; hence the cube of the diameter in inches divided by 529·554 gives the quotient in imperial gallons; and the cube of the diameter in inches divided by 4236·43 gives the quotient in imperial bushels.

The *gauge points* for a sphere are $\sqrt{529\cdot 554}$ and $\sqrt{4236\cdot 43}$, or 23·0 and 65·1, which are respectively the gauge points for gallons and bushels.

EXAMPLES IN FINDING THE AREAS OF SUPERFICIES.

I. A SQUARE.

RULE.

Multiply the side into itself, and divide the product by 277·274 for imperial gallons, and by 2218·192 for imperial bushels.

EXAMPLE.

If a side of a square be 40 inches, required its area in imperial bushels.

$40^2 = 1600.$

277·3)1600·000(5·77, the area, nearly.
1386 5
21350
19411
19390

Or, 2218·2)1600·00(·7213 imperial bushels.
155274
47260
44364
28960
22182
6778

By the Slide Rule.

	On A.		*on B.*		*On A.*		*on B.*	
As	277·3	:	40	: :	40	:	5·77	= the area.

Or thus, by the Gauge Point.

	On D.		*On C.*		*On D.*		*On C.*	
As	16·65	:	1	: :	40	:	5·77	= the area.

ıis position of the rule, any side of a square on D, shows ·responding area on C.

II. A PARALLELOGRAM.

RULE.

tiply the length by the breadth, and divide the product ·274 for imperial gallons, and by 2218·192 for imperial).

EXAMPLE.

length and breadth of a rectangular parallelogram are 30 inches respectively, find its area in imperial gallons.

```
              50
              30
277·3)15000(5·41 area nearly in imperial gallons,
      13865        or = ·6762 imperial bushels.
      11350
      11092
       2580
```

By the Slide Rule.

	On A.		*on B.*		*On A.*		*on B.*	
As	277·3	:	50	: :	30	:	5·4	the area.

III. A RHOMBUS.

RULE.

;iply the base by the perpendicular altitude, and divide duct as before for gallons and bushels.

EXAMPLE.

le of a rhombus = 40 inches, and the perpendicular al = *37 inches*; find its area in imperial gallons.

$$\begin{array}{r} 37 \\ 40 \\ \hline \end{array}$$

$$277{\cdot}3)1480{\cdot}0(5{\cdot}337 \text{ the area in gallons}$$

$$\begin{array}{r} 13865 \\ \hline 9350 \\ 8319 \\ \hline 10310 \\ 8319 \\ \hline 19910 \\ \hline \end{array}$$

And $\dfrac{1480}{2218{\cdot}192} = {\cdot}667$, the area in bushels.

By the Slide Rule.

On A. on B. on A. on B.
As 277·3 : 40 : : 37 : 5·33, the area as befo
This rule applies to the rhomboid.

IV. A TRIANGLE.

RULE.

Half the base multiplied by the perpendicular wi product, which being divided by 277·3 or by 2218·92 the area in bushels or gallons.

EXAMPLE.

If the base of a triangle be 60 inches, and the perp altitude 23·5 : what is the area in imperial gallons?

$$\begin{array}{r} 23{\cdot}5 \\ 30 = \frac{1}{2} \text{ base.} \\ \hline \end{array}$$

$$277{\cdot}3)705{\cdot}00(2{\cdot}54 = \text{area in gallons}$$

$$\begin{array}{r} 5546 \\ \hline 15040 \\ 13865 \\ \hline 1175 \\ \hline \end{array}$$

By the Slide Rule.

On A. on B. on A. on B.
As 277·3 : 30 :: 23·5 : 2·54 the area as before.

V. A TRAPEZIUM.

RULE.

aw one of the diagonals of the trapezium, and let fall two ndiculars upon it from the opposite angles; then one-half ım of these perpendiculars being multiplied by the diagonal give the area required.

EXAMPLE.

e diagonal of a trapezium is 60 inches, and the two per-culars upon it are 15 and 27 inches: find the area in im- gallons.

```
          15
          27
          --
        2)42
          --
          21
           60
      ------
277·3)1260·000(4·54 = area in im. galls.
      11092
      -----
       15080
       13865
       -----
        12150
        11092
        -----
         1058
```

By the Slide Rule.

On A. on B. on A. on B.
As 277·3 : 42 :: 30 : 4·54 the area as before.

VI. A CIRCLE.

RULE.

Divide the square of the diameter by 353·036 for i gallons, and by 2824·288 for imperial bushels.

EXAMPLE.

The diameter of a circle is 45 inches ; find its area in rial gallons.

```
          45
          45
        ----
         225
        180
     -------
353·036)2025·00000(5·736 = the area in i
        1765180
        -------
         2598200
         2471252
         -------
          126948
          ------
```

By the Slide Rule.

On B. *on A.* *on B.* *on A.*
As 353 : 45 : : 45 : 5·74

Or :

On D. *on C.* *on D.* *on C.*
As 18·79 : 1 : : 45 : 5·74 = the area.
Cir. gauge point.

EXAMPLES IN FINDING THE CONTENTS OF SOLIDS

I. A CUBE.

Cube the given side in inches, and divide by 277·3 f ns, and by 2218·2 for imperial bushels.

EXAMPLE.

The side of a cube is 60 inches; find its content in imperial llons and bushels.

$$60^3 = 216000 = \text{content in inches.}$$

$$\text{And } \frac{216000}{277 \cdot 3} = 779 \text{ gallons, nearly.}$$

$$\text{Also } \frac{216000}{2218 \cdot 2} = 97 \cdot 4 \text{ bushels.}$$

By the Slide Rule.

	On D.		*on C.*		*on D.*		*on C.*
As	16·65	:	60	: :	60	:	779
	On D.		*on C.*		*on D.*		*on C.*
And, as	47·1	:	60	: :	60	:	97·4

here 16·65 and 47·1 are the square roots of 277·3, 2218·2, spectively.

II. A CYLINDER.

RULE.

The continued product of the altitude, square of base diater, and ·7854, will give the content in cubic inches; and s being divided by 277·3 and 2218·2, will give the content imperial gallons and bushels.

EXAMPLE.

If the altitude of a cylinder be 30 inches, and its base diater 24 inches, find its content in imperial gallons.

```
      24² — 576
              30 = alt.
           -----
           17280
           ·7854
           -----
           69120
          8640
        13824
       12096
277·3)13571·7120(48·9 = content in imperial gallons.
       11092
       -----
        24797
        22184
        -----
         26131
        ------
```

By the Slide Rule.

On D.		*on C.*		*on D.*		*on C.*
As 18·79	:	30	: :	24	:	48·9 as befo

Gauge point for cir.

III. A CONE.

RULE.

Multiply the square of the base diameter by $\frac{1}{3}$ of t and the product divided by 353 will give the quotie perial gallons. If the cone be an oblique one, who an ellipse, multiply the transverse, and conjugate together, and multiply the product by $\frac{1}{3}$ of the height result, divided by 353, will give the quotient i gallons.

EXAMPLE.

The base diameter of a cone is 38 inches, and it 45 inches; what is the content in imperial gallons?

$38^2 = 1444$

$15 = \frac{1}{3}$ the altitude.

353)21660(61·35 = content in imperia
2118

480
353

1270
1059

2110
1765

345

By the Slide Rule.

	on D.		on C.		on D.		on C.
As	18·79	:	15	: :	38	:	61·35 content as before.

IV. A SPHERE.

RULE.

Multiply the cube of the diameter by ·5236, and the product is the content in inches: this product divided by 277·3, will give the quotient in imperial gallons; and divided by 2218·2, will give the content in imperial bushels.

Or,

The cube of the diameter multiplied by ·0018884 will give the content in imperial gallons.

Or,

The cube of the diameter divided by 529·55, will give the content in imperial gallons; divided by 4236·43, will give the content in imperial bushels.

EXAMPLE.

The diameter of a sphere is 34 inches; find its content in imperial gallons.

$$\text{Content} = \frac{\cdot 5236}{277\cdot 3} \times 34^3 = \frac{24^3}{529\cdot 554} = 74\cdot 22 \text{ imperial gal.}$$

$$\text{Also content} = \frac{\cdot 5236}{2218\cdot 2} \times 34^3 = \frac{34^3}{4236\cdot 43} = 9.28 \text{ imp. bush.}$$

By the Slide Rule.

	on D.		on C.		on D.		
As	23·0	:	34	: :	34	:	74·22 imperial gallons.
Gauge point for gall.							
	65·1	:	34	: :	34	:	9·28 imperial bushels.
Gauge point for bush.							

V. A PARABOLIC SPINDLE.

RULE.

Multiply the square of the middle diameter by the length of the spindle; and divide the product by 661·94 for imperial gallons, and by 5295·54 for imperial bushels.

Since the content of a parabolic spindle $= \frac{8}{15}$ of circumscribing cylinder $=$ circumscribing cylinder $\div \frac{15}{8} =$ circumscribing cylinder $\div$ 1·875, therefore the circular divisors multiplied by 1·875 give divisors for a parabolic spindle.—Hence,

$$\left.\begin{array}{r} 353{\cdot}036 \times 1{\cdot}875 = 661{\cdot}943 \\ \text{and } 2824{\cdot}29 \times 1{\cdot}875 = 5295{\cdot}54 \end{array}\right\} \text{which are divisors for a}$$

parabolic spindle in gallons and bushels respectively.

Also,

$\sqrt{661{\cdot}943}$ and $\sqrt{5295{\cdot}54}$, *i. e.* 25·73 and 72·77 are the guage points for gallons and bushels respectively.

PROBLEM.

To find the content of a cask, having given the head, middle, and bung diameters, and the length of the cask.

RULE I.

To the square of twice the middle diameter, add the squares of the head and bung diameters; multiply the sum by $\frac{1}{6}$th of the length, and this product divided by 353·04 will give the content in imperial gallons.*

EXAMPLE.

The length of a cask is 40 inches, the bung diameter being 32, the head diameter 24, and the middle diameter between the bung and the head 28·75 inches; find its content in imperial gallons.

* See the demonstration of Problem XXX. in the Appendix to the Mensuration. This rule is one of the most accurate of all the rules findi[ng the c]ontents of casks; and applies to the four varieties with

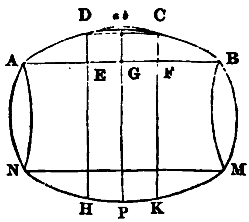
$$32^2 = 1024 = (\text{bung diameter})^2$$
$$24^2 = 576 = (\text{head diameter})^2$$
$$(57{\cdot}5)^2 = 3306{\cdot}25 = \text{square of double the diameter in the middle.}$$

```
 4906·25
  6·6666 = 1/6 length.
 -------
   2943750
  2943750
 2943750
2943750
2943750
-----------
353·04)32708·006250(92·646 imperial gallons.
       317736
       ------
         93440
         70608
         ------
        228326
        211824
        ------
         165022
         141216
         ------
          238065
          211824
          ------
           26241
          ------
```

RULE II.

l together 39 times the sqaare of the bung diameter, 25 *he square of the* head diameter, and 26 times the pro- *the diameters;* multiply this sum by the length, an *product by* ·00031, and the result will be the cont *'al gallons.*

In the investigation of this rule it is supposed that tl dle part of the cask, extending to $\frac{1}{3}$ of the whole, is the frustum of a parabolic spindle, and that the two ends ı conical frustums, equal to one another, the length (being $\frac{1}{3}$ the length of the cask.



Let A D, C B be the two parts which are considere rectilineal, and D b C the curved part. Draw D E H perpendicular to A B, and meeting the lower side of tl in H, K.—Produce A D, B C to meet in a, then are D tangents to the parabola. Draw a b o G perpendicular Then $a\,o$ is the subtangent, and $b\,o$ is the abscissa; th from the nature of the parabola, $a\,o = 2\,o\,b$, or $b\,o =$

Now since A E $=$ E F $=$ F B, each being $\frac{1}{3}$ of A I therefore, D $o = \frac{1}{3}$ A G, and $a\,o = \frac{1}{3}\,a$ G, consequen $= \frac{1}{6}\,a$ G $= \frac{1}{5}\,b$ G $= \frac{1}{10}\,(b$ P $-$ A N$)$.

Suppose $\Delta =$ bung diameter b P.

$\delta =$ end diameter A N.

$\lambda =$ A B the length of the cask.

$$\therefore \text{D H} = b\,\text{P} - 2\,b\,o = \Delta - \tfrac{1}{5}(\Delta - \delta) = \quad 4$$

Now by problem 29 in the mensuration, for finding t tent of the middle frustum of a parabolic spindle, we fi

$$solid\ \text{D H K C} = \frac{8\,\Delta^2 + 4\,\Delta . \frac{4\,\Delta + \delta}{5} + \frac{3}{25}(4\,\Delta + \delta)^2}{15}$$

$$= \frac{200\,\Delta^2 + 80\,\Delta^2 + 20\,\Delta\,\delta + 48\,\Delta^2 + 24\,\Delta\,\delta +}{25 \times 15}$$

$$= \frac{328\,\Delta^2 + 44\,\Delta\,\delta + \delta^2}{25 \times 45} . \pi\,\lambda \qquad (\text{A}).$$

Also the two conical frustums D A N H, C K M B

$$= \frac{\frac{1}{25}(4\,\Delta + \delta)^2 + \frac{1}{5}(4\,\Delta + \delta)\,\delta + \delta^2}{3} . \frac{2\,\pi\,\lambda}{3}$$

$$= \frac{2\,(4\,\Delta + \delta)^2 + 10\,(4\,\Delta + \delta)\,\delta + 50\,\delta^2}{9 \times 25} . \pi\,\lambda$$

$$= \frac{10\,(4\,\Delta + \delta)^2 + 50\,(4\,\Delta + \delta)\,\delta + 250\,\delta^2}{45 \times 25} . \pi\,\lambda$$

$$= \frac{160\,\Delta^2 + 280\,\Delta\,\delta + 310\,\delta^2}{45 \times 25} . \pi\,\lambda \qquad (\text{B}.)$$

$\therefore$ the content of the whole cask

$$= \text{A} + \text{B} = \{488\,\Delta^2 + 324\,\Delta\,\delta + 313\delta^2\} . \frac{\pi\,\lambda}{1125}$$

$$= \{39\,\Delta^2 + 26\,\Delta\,\delta + 25\,\delta^2\} . \frac{\pi\lambda}{90} \text{ cubic inches.}$$

But $\frac{\pi}{90} = \frac{\cdot 785398}{90}$ and $\frac{\cdot 785398}{90 \times 277 \cdot 274}$

$= \cdot 000031$; hence the content

$= \{39\,\Delta^2 + 26\,\Delta\,\delta + 25\,\delta^2\} \times \lambda \times \cdot 000031$ imperial galls.

I consider this rule to be very exact.

MISCELLANEOUS PROBLEMS.

1. There is a rectangular deal plank whose length is 7 fee 3 inches, and breadth 18 inches. A sawyer wishes to cut *of the whole; how far from one end must his saw pass, sing him to cut through one of the angles?*

	feet.	in.	
Length =	7	3	
Breadth =	1	6	
	7	3	
	3	7	6
Area of the plank =	10	10	6
3)			
$\frac{1}{3}$ of the plank =	3	7	6

Now it is evident that the piece cut off must be triar and its area equal to 3 feet, 7 in., 6 pts.

foot. in. feet. in. parts.

Hence $\frac{1}{2}$ of (1 6) : (3 7 6) : : 1 : length requ

feet long. sq. feet.

Or ·75 : 3·625 : : 1 : length required.

1

·75)3·625(4·8333 feet = 4 feet, 10 inche
300

625
600

250
225

250

2. Find the difference between the area of a triangle sides are 3, 4, and 5 feet; and the area of an equilater angle having an equal perimeter.

Perimeter of the equilat. △ = 12 ; therefore each sid feet ; and its area = $\frac{1}{2}$ base × perp. = $\frac{1}{2}$ × 4 × $\sqrt{4^2-}$ $2\sqrt{12}$ = 6·928 sq. feet. And the area of the triangle three sides are 3, 4, and 5 feet, is equal to $\sqrt{6 \times 3 \times}$ = *sq. feet.*

Hence the difference = 6·928—6 = ·928 of a sq.

3. The breadth of a plank is 26 inches; what must be the gth of the part cut off, so that it may contain 1 yard, 4 feet, inches, superficial measure?

yd.	*ft.*	*in.*		*sq. in.*
1	4	72	=	1944

26)1944(74·7 = 6 feet $2\frac{7}{60}$ inches = the length req.
182
124
104
200
182
18

4. If the diagonals of a quadrilateral figure be 18 and 12 t, and the angle at which they cut one another be 60°; find e area of the figure.

The area of any quadrilateral is equal to the product of its o diagonals multiplied by half the sine of the angle formed by eir intersection. (See the 1st Problem in the miscellaneous llection at the end of the Appendix to the Mensuration.)

ence the area required $= 18 \times 12 \times \frac{1}{2} \sin 60°$
$= 108 \cdot \sqrt{\frac{3}{2}} = 54\sqrt{3} = 93{\cdot}528$ sq. feet.

5. A joist is $8\frac{1}{2}$ inches deep and $3\frac{1}{2}$ inches broad: find the mensions of a scantling twice as big as the joist; the breadth the scantling being $3\frac{3}{4}$ inches.

Depth $= 8\frac{1}{2} = 8{\cdot}5$
Breadth $= 3\frac{1}{2} = 3{\cdot}6$

425
255

rea *of one end of the joist* $= 29{\cdot}75$

```
           29·75
               2
           -----
     3·75)59·50(15·86 inches, the
          375                    [the s
          ----
          2200
          1875
          ----
           3250
           3000
           ----
            2500
            ----
```

6. A cistern is to be lined with lead weighing 7 lt
square foot; what will be the cost thereof, supposing t
of the cistern to be 4 feet 6 inches, and the side of it
base 2 feet 9 inches; the value of lead being $4\frac{1}{2}d.$ per

```
         feet. in.
           2    9
                4
         --------
          11    0
           4    6
         --------
          44    0
           5    6
         --------
          49    6 = area of the four s

       feet. in.
         2    9
         2    9
       --------
         5    6
         2    0   9
       ------------
Area of base = 7    6   9
              49    6   0
             -------------
              57    0   9
```

	feet.	in.	pts.
	57	0	9
			7
	399	5	3
			4
	1597	9	0
$\frac{1}{2}$ is $\frac{1}{8}$ of 1*d*.	199	8	$7\frac{1}{2}$
	12)1797	5	$7\frac{1}{2}$
	20)149	$9\frac{1}{2}$ nearly.	
	£7	9	$9\frac{1}{2}$ nearly: the answer.

7. Two circles touch each other internally, and the area of e lune cut out of the larger circle, is equal to twice the area the smaller circle. Determine the radius of the smaller circle, pposing the radius of the larger circle = 5 feet.

Let x = radius of smaller circle.

$x^2 : 5^2 : :$ area of small cir. : area of large cir.

And $x^2 : 5^2 - x^2 : :$ area of small cir. : area of lune.

But area of the lune = twice the area of the smaller cir.

$\therefore\ 5^2 - x^2 = 2\,x^2$

and $5^2 = 3\,x^2$ and $x = \dfrac{5}{\sqrt{3}} = 2{\cdot}88$ feet.

8. A rectangular flat roof is 24 feet 8 inches by 14 feet 6 ıches; what will be the cost of covering it with lead at 28s. ər *cwt., supposing* the thickness of the lead to be such as to əigh *8 lbs. to* the square foot?

feet.	*in.*	
24	8	
14	6	
345	4	
12	4	
357	8	
	8	lbs. per sq. foot.
2861	4	
	3*d.*	per lb.
12)8584	0	
20)715	4	
£35 15	4	

9. If the surface of a sphere be represented by the number 4: then the circumscribed cylinder's convex surface, and whole surface, will be 4 and 6. Also the circumscribed equilateral cone's convex and whole surface will be 6 and 9 respectively.

Suppose δ = diameter of the sphere.

$\pi = \cdot 7854$

Then $4\,\pi\delta^2$ = surface of the sphere.

The circumscribed cylinder's convex surface = circumf. of the base $\times$ altitude $= 4\,\pi\delta \times \delta = 4\,\pi\delta^2$, the same expression as for the surface of the sphere.

Also the *whole* surface of the cylinder $= 4\,\pi\delta^2$ + area of the two ends $= 4\,\pi\delta^2 + 2\,\pi\delta^2 = 6\,\pi\delta^2$.

Again, the circumscribed equilateral cone's convex surface = circumf. of the base $\times$ $\frac{1}{2}$ slant side.

Now from the nature of the figure the semi-base diameter of the equilateral cone $= \frac{\delta}{2}\sqrt{3}$ therefore the diameter of the cone's base $= \delta\sqrt{3}$; and the circumference $= 4\,\pi\delta\sqrt{3}$; also the slant side of the cone $= \delta\sqrt{3}$; hence the convex surfa[illegible] cone $= 4\,\pi\delta\sqrt{3} \times \frac{\delta}{2}\sqrt{3} = 6\,\pi\delta^2$

Lastly, the whole surface of the cone = its convex surface + the area of its base $= 6\ \pi\delta^2 + 3\ \pi\delta^2 = 9\ \pi\delta^2$.—These various expressions for the surfaces, show that their values are as the integers stated in the Problem.

10. There is an equilateral triangular court-yard the paving of which cost as much at 8*d*. per square foot, as the enclosing of the same at one guinea per yard, running measure. What is the length of a side of this triangle?

Let x = a side of this equilateral triangle in feet.

$$\text{The area} = \left\{\frac{3\ x}{2}\left(\frac{3\ x}{2}-x\right).\left(\frac{3\ x}{2}-x\right).\left(\frac{3\ x}{2}-x\right)\right\}^{\frac{1}{2}}$$

$$= \left(\frac{3\ x}{2}.\frac{x}{2}.\frac{x}{2}.\frac{x}{2}\right)^{\frac{1}{2}} = \frac{x^2\sqrt{3}}{4} \text{ square feet.}$$

The cost of which at 8*d*. per foot $= 2x^2\sqrt{3}$ pence.

The palisading of one side of the triangle cost $\frac{x}{3}$ guineas, and of enclosing the triangle the cost is x guineas, or 252 x pence; because three feet cost one guinea.

$$\therefore 2\ x^2\ \sqrt{3} = 252\ x$$

$$\text{Or } x = \frac{126}{\sqrt{3}} \text{ feet} = \frac{126}{1{\cdot}732} = 72{\cdot}74 \text{ feet.}$$

11. A gentleman wishes to dig a circular fish-pond in his garden; what will be the breadth of the pond, supposing it to occupy three quarters of an acre?

Let δ = pond's diameter, in yards.

Then $\cdot 7854\ \delta^2 = \frac{3}{4}$ of an acre $= \frac{3}{4}$ of 4840 sq. yards.

$$\therefore \delta^2 = \frac{3630}{\cdot 7854} = 4621{\cdot}85$$

$$\therefore \delta = 67{\cdot}98 \text{ yards.}$$

12. The four sides of a field whose diagonals are equal to each other, are 25, 35, 31, and 19 poles respectively; how many acres does the field contain?

Now in every quadrilateral figure the rectangle contained by *the diagonals is* equal to the sum of the rectangles contained by *the opposite sides*; (Euc. b. 6. prop. D.) hence if x = one of *the diagonals*, we have

15. The distance between the centres of two circle inches; what is the area of the space inclosed by the ferences, supposing the diameters of the circles are ea to 50 inches?

The versed sines of the two equal arcs which bound t whose area is required, are each $=$ 10 inches; hence $\frac{1}{5}$ $=$ tab. vers. sine.

And tab. area corr. to $\cdot 2 = \cdot 111823$

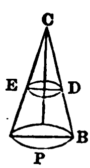
$$\begin{array}{r} 2500 = 50^2 \\ \hline 55911500 \\ 223646 \\ \hline \end{array}$$

One half the required area $= 279\cdot557500$

$$\begin{array}{r} 2 \\ \hline \end{array}$$

Space required contains $559\cdot115$ sq. inches.

16. Two townships, A and B, having but one church bute to the repairs thereof in the proportion of 4 to 5 happens that the church has a conical wooden spire whi the protection of a little paint. The circumference of th base is 47 feet, and the altitude is 20 yards. Now if th ship A paints the upper part of the spire with black p the township B paints the lower part with white paint, from the spire's base will be the boundary between the lours; the price of white paint being twice as great as of black paint?



Let A C B be the conical spire. Then since the circ of the cone's base $=$ 47 feet, therefore the diameter

$$= \frac{47}{3\cdot1416} = 14\cdot9605 \text{ feet}; \text{ and the radius A P} =$$

Now C P $= 20$ yards $= 60$ feet.

$\therefore$ C A $= \sqrt{60^2+(7\cdot4802)^2} = \sqrt{3655\cdot95339204} = 60\cdot4644$

And surface of whole cone $= 47 \times 30\cdot2322 = 1420\cdot9134$

If C E $= x$,

we have circumf. A B : circumf. E D :: C A : C E.

or 47 : circumf. E D :: 60·4644 : x.

$$\therefore \text{ Circumf. ED} = \frac{47x}{60\cdot4644}$$

$$\therefore \text{ Surface of CED} = \frac{47x}{60\cdot4644} \times \frac{x}{2} = \frac{47x^2}{120\cdot9288}$$

$$\therefore \text{ Surface of the frustum EABD} = 1420\cdot9134 - \frac{47x^2}{120\cdot9288}$$

$\therefore$ cost of painting the upper part with black paint : cost of painting the lower part with white paint :: $\dfrac{47x^2}{120\cdot9288} \times 1$: $\left\{1420\cdot9134 - \dfrac{47x^2}{120\cdot9288}\right\} \times 2$

But these charges are as 4 : 5

$$\frac{47x^2}{120\cdot9288} \times 1 : \left\{1420\cdot9134 - \frac{47x^2}{120\cdot9288}\right\} \times 2 :: 4 : 5$$

$$\therefore \frac{235x^2}{120\cdot9288} = 11367\cdot3072 - \frac{376x^2}{120\cdot9288}$$

$$\therefore \frac{611x^2}{120\cdot9288} = 11367\cdot3072$$

Whence $x = 47\cdot43 =$ C E

$\therefore 60\cdot4644 - 47\cdot43 = 13\cdot0344$ feet from the base: hence the township B must paint the lower part of the conical spire, to the distance of 13·0344 feet up the slant height; and the township A must paint the remaining part of the spire.

Otherwise,

Since the circumference of the cone's base is given, suppose $= c$; also let $\sigma =$ C A $=$ slant side of the cone; then if the diameter A B $= b$, and $\pi = 3\cdot1416$ and $x =$ C E, we have

$$\sigma : b :: x : \text{ED} = \frac{b\,x}{\sigma}$$

and $\dfrac{\pi b x^2}{2\sigma} =$ area of convex surface C E D

[illegible] represent the proportional [illegible] [illegible] the upper and lower parts: and by condition [illegible]

[illegible] whence $x = 47.41$ feet.

Otherwise.

Since [illegible] are similar

Surface [illegible] : surface [illegible] :: [illegible] $:: r^2 : x^2$

[illegible] surface [illegible] : surface [illegible] $:: r^2 - x^2 : x^2$

and cost of painting [illegible] : ditto [illegible] $:: [illegible] (r^2 - x^2) : x^2$

[illegible]

$$\text{whence } x = 3\,r\sqrt{\frac{[illegible]}{13}}$$

[illegible] we have $\frac{47}{3\cdot1416} = 14\cdot9605 = \text{AB}$.

Whence [illegible] $= 7\cdot4802$ feet, and [illegible] $= 60$ feet; whence [illegible] $= 50\cdot4644$ feet, and $x = 47\cdot4[illegible]$

[illegible]

A cable which is 3 feet long, and 9 inches in circumference, weighs 22 lbs., determine the weight of that cable whose diameter is [illegible] inches, and length 1 fathom.

[illegible] the weight varies as the magnitude, since the density [illegible]

$[illegible] \times \cdot07958 \times 3 : 9^2 \times \cdot7854 \times 6$

$\cdot07958 : 1\cdot5708$

$$\frac{1\cdot5708}{\cdot07958} \times 22 = 434\cdot25 \text{ lbs.}$$

[illegible] small cubes, a side of which $= 4$ inches, [illegible] large cube, whose side is 8 inches?

$8^3 = 512$

$4^3 = 64$

[illegible] the number of small cubes required.

[illegible] place an oaken curb to a [illegible] what will be the expe[illegible]

of, supposing the breadth of the curb to be $7\frac{1}{4}$ inches, and the inner diameter to be 3 feet 6 inches?

It is here supposed that the curb is annular; consequently the content of the oak $= \{(56\cdot5)^2 - 42^2\} \times \cdot7854 = \{3192\cdot25 - 1764\} \times \cdot7854 = 1428\cdot25 \times \cdot7854 = 1121\cdot74755$ square inches $= 7\cdot789$ square feet, which at $10d.$ per square foot $= 77\cdot89$ pence $= 6s.\ 5\frac{3}{4}d.$ the expense required.

20. A paviour charged £10 for paving a semicircular court-yard at $2s.\ 6d.$ per square foot; what is the diameter of the pavement?

Let $\delta =$ diameter of the pavement in feet.

Then $\frac{1}{2}\ \delta^2 \times \cdot7854 \times 30d. =$ cost of paving the semicircular yard at $30d.$ per square foot.

Hence $\frac{1}{2}\ \delta^2 \times \cdot7854 \times 30 = 2400$ pence.

$$\text{Or } \delta^2 \times \cdot7854 = 160$$

$$\therefore\ \delta^2 = \frac{160}{\cdot7854} = 203\cdot7178$$

$$\text{and } \delta = 14\cdot27 \text{ feet.}$$

21. A farmer borrowed of his neighbour part of a hay rick, which measured 6 feet in length, breadth, and thickness. At the next hay time he paid back two equal cubic pieces each side of which was four feet. Has the debt been discharged?

$$4^3 = 64$$
$$\underline{\quad 2}$$

128 cubic feet = the content of the hay which was returned. But $6^3 = 216$ cubic feet = the content of the hay which was borrowed; hence $216 - 128 = 88$ cubic feet the content of the hay which remains unpaid, and which is therefore due to the lender.

22. What is the worth of a piece of oak timber of uniform thickness; whose end is an ellipse, the major and minor axes of which are 3 and 2 feet respectively; the length of the timber being 20 feet, and its value $2s.\ 3d.$ per cubic foot?

The area of the elliptic end $= 3 \times 2 \times \cdot7854 = 4\cdot71$

Hence 4·7124
20

Content = 94·2480 cubic feet.
27 pence per cubic foot.

659736
188496

12)2544·696 pence.

2,0)21,2·058

£10 12*s*. 0½*d*. = value required.

23. Three men bought a grindstone, whose diameter was 60 inches: what part may each person grind away, supposing the men to have paid equal shares of the cost?

See the figure in Problem XLVIII. of the Practical Geometry. Let O A be the radius of the grindstone; then make A D = D E = E O = 10 inches; upon O A describe a semi-circle, and draw D H, E I perpendicular to O A; then if circles be described with radii O H, O I, the parts due to each purchaser will be marked out. For O A : O H :: O H : O D; hence O A : O D :: O A^2 : O H^2 :: cir. A B C : cir. des. with radii O H. ∴ O A : A D :: O A^2 : O A^2 — O H^2 :: cir. A B C : first annulus.

i. e. 3 : 1 :: cir. A B C : first annulus.
∴ first annulus = $\frac{1}{3}$ the cir. A B C.

In a similar manner, it may be shown that the annulus contained between the circumferences of the circles described with radii O H, O I is equal to $\frac{1}{3}$ of the whole circle A B C; and consequently the circle described with radius O I contains $\frac{1}{3}$ part of the whole circle.

To compute the values of (O A — O H), (O H — O I), or the breadths of these annuli, we have

O A : O H :: O H : O D
30 : O H :: O H : 20
∴ O H = $\sqrt{600}$ = 24·494.

Hence 30 — 24·494 = 5·506 = breadth of the exteric annulus.

Again, O A : O I : : O I : O E.

Or, 30 : O I : : O I : 10.

∴ O I = $\sqrt{300}$ = 17·320 = radius of the innermost part

Consequently O H — O I = 24·494 — 17·320 = 7·174 = breadth of the second annulus. The person who first uses th grindstone may therefore wear away 5·506 inches; the secon person may wear away 7·174 inches; and the last person' share will be a circle, whose radius is 17·320 inches.

24. How many gallons, imperial measure, are contained i a cistern, whose length and breadth at the top are 5 and 4 fee respectively; and at the bottom 4 and 3 feet: the perpen dicular depth being $3\frac{1}{2}$ feet?

Area of one end = 20 square feet.
Area of other end = 12 square feet.
4 times area of mid. sect. = 63 square feet.

95
·5833 = $\frac{1}{6}$ the height.

29165
52497

55·4135 = content in cubic feet.

Now one gallon imperial measure contains 277·274 cubi inches, and one cubic foot contains 1728 cubic inches, ∴ on cubic foot contains $\frac{1728}{277.274}$ gallons, or 6·232 gallons; henc if cubic feet be multiplied by 6·232, the result will be in im perial gallons.

55·4135
6·232

1108270
1662405
1108270
3324810

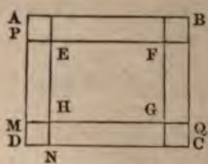
Content = *345·3369320* gallons.

25. A ladder 30 feet long, may be so placed that it shall reach a window 20 feet from the ground, on one side of the street; and by turning it over, without removing the foot, it will reach a window 24 feet high, on the other side of the street. What is the breadth of the street?

From the nature of this problem, it is evident that the breadth of the street is equal to the sum of the bases of two right angled triangles, whose perpendiculars are 20 and 24, the hypothenuse 30 being common to both: hence the required breadth $= (30^2-20^2)^{\frac{1}{2}}+(30^2-24^2)^{\frac{1}{2}}$

$$=\sqrt{500}+\sqrt{324} = 22{\cdot}36+18 = 40{\cdot}36 \text{ feet.}$$

26. A gentleman having a garden in the form of a rectangle, 100 feet long, and 80 feet broad, wishes to make a gravel walk round it. Determine the breadth of the walk, so that it shall just take up one-sixth part of the ground?



Let A B C D be the rectangular garden; and let $x =$ H N $=$ the breadth of the walk.

Then $100\ x =$ area of parallelogram M C.

$100\ x =$ area of parallelogram P B.

Also P M $= 80 - ($A P$+$D M$) = 80 - 2\ x$.

$\therefore$ Parallelograms P H, F Q together $= (80 - 2\ x) \times 2\ x$.

Also area of the whole garden $= 100 \times 80 = 8000$.

Hence $200\ x + (80 - 2\ x)\ .\ 2\ x = \frac{1}{6}$ of $8000 = 1333{\cdot}33$.

Or $200\ x + 160\ x - 4\ x^2 = 1333{\cdot}33$.

Whence $4\ x^2 - 360\ x = -1333\frac{1}{3}$.

$x^2 - \ 90\ x = - \ 333\frac{1}{3}$.

$x^2 - \ 90\ x + 45^2 = 45^2 - 333\frac{1}{3} = 1691{\cdot}6666$.

$\therefore x - 45 = \sqrt{1691{\cdot}6666} = \pm\ 41{\cdot}13$ feet nearly.

$\therefore x = 45 + 41{\cdot}13 = 3{\cdot}87$; where it is evident that we must take the negative value of $41{\cdot}13$, or we should obtain x greater than the breadth of the garden.

27. Determine the diameter of a circle whose area is equal to the convex surface of a cone whose altitude is 5 feet, and the diameter 3 feet.

Radius of cone's base $= 1\cdot5$ feet.

$\because$ slant side of cone $= \{5^2+(1\cdot5)^2\}^{\frac{1}{2}} = 5\cdot22$ feet.

Also the circumference of the base $= 3\cdot1416\times3 = 9\cdot4248$.
$\therefore$ convex surface of the cone $= 9\cdot4248\times2\cdot61$.
$= 24\cdot598728$.

Hence if $x =$ diameter of the required circle.

$$\cdot7854\times x^2 = 24\cdot598728.$$

$$\therefore x^2 = \frac{24\cdot598728}{\cdot7854} = 31\cdot32.$$

and $x = \sqrt{31\cdot32} = 5\cdot6$ feet nearly.

28. It is required to determine the thickness of the lead in a pipe of $1\frac{1}{2}$ inches bore; supposing a cubic foot of lead to weigh 11325 ounces, and the pipe itself to weigh 18 lb. per running yard.

Since 1 cubic foot contains 11325 oz.

$\therefore$ 1 oz. contains $\frac{1728}{\cdot11325}$ cubic inches.

And 288 oz. or 18 lbs. contain $\frac{1728\times288}{\cdot11325}$ cubic inches.

Now if $x =$ the thickness of the lead,
$\{(2x+1\cdot5)^2-(1\cdot5)^2\}\times36\times\cdot7854 =$ the content of 1 yard of the pipe in cubic inches; but 1 yard of the pipe weighs 18 lbs. and contains $\frac{1728\times288}{11325}$ cubic inches; hence

$$\{(2\,x+1\cdot5)^2-(1\cdot5)^2\}\times36\times\cdot7854 = \frac{1728\times288}{11325}$$

$$\text{Or } (4\,x^2+6\,x)\times\cdot7854 = \frac{48\times288}{11325} = 1\cdot220662$$

$$\therefore 4\,x^2+6 = \frac{1\cdot220662}{\cdot7854} = 1\cdot5542 \text{ nearly.}$$

$$x^2+\tfrac{3}{2}\,x = \cdot3885$$

$$x^2+\tfrac{3}{2}\,x+\tfrac{9}{16} = \cdot5625+\cdot3885 = \cdot9510$$

$$x+\tfrac{3}{4} = \cdot975$$

$x = \cdot975-\cdot75 = \cdot225$ of an inch $=$ the thickness re

29. Determine the areas of an equilateral triangle, a sq
a hexagon, dodecagon, and a circle; the perimeter of eac
ing 49 feet.

First to find the area of the triangle.

$\frac{1}{2}$ perimeter = 24·5
and each side = 16·333

24·5	has for its log.	1·3891661
8·167	has log.	0·9120626
8·167	log.	0·9120626
8·167	log.	0·9120626
		2)4·1253539
115·55 =		2·0626769

area of the triangle.

A side of the square = 12·25

$$\begin{array}{r} 12\cdot25 \\ 12\cdot25 \\ \hline 6125 \\ 2450 \\ 2450 \\ 1225 \\ \hline 150\cdot0625 \end{array}$$

Area of the square = 150·0625

A side of the hexagon = $\frac{49}{6}$ = 8·1666.
Tabular multiplier is 2·598056
66·6933 = $(8\cdot1666)^2$ near

$$\begin{array}{r} 7794228 \\ 7794228 \\ 23382684 \\ 15588456 \\ 15588456 \\ 15588456 \\ \hline 173\cdot2742610908 \end{array}$$

Area of hexagon = 173·2742610908

A side of the duodegon $\frac{49}{12} = 4{\cdot}0833$.

Tabular multiplier is $11{\cdot}196152$

$$
\begin{array}{r}
16{\cdot}6733 = (4.0833)^2 \text{ nearly.} \\
\hline
33588456 \\
33588456 \\
78373064 \\
67176912 \\
67176912 \\
11196152 \\
\hline
\end{array}
$$

a of duod. $= 186{\cdot}6768011416$

For the circle, $\cdot 07958$

$$
\begin{array}{r}
\cdot 07958 \\
2401 = 49^2 \\
\hline
7985 \\
31832 \\
15916 \\
\hline
\end{array}
$$

Area of a circle $= 191{\cdot}07158$

0. What will be the cost of gilding the ball at the top of St 's Cathedral, at the rate of 4*d*. per square inch; the diaer of the ball being 6 feet?

$$
\begin{array}{r}
\cdot 7854 \\
36 = 6^2 \\
\hline
47124 \\
23562 \\
\hline
28{\cdot}2744 = \text{area of a great cir. of this sph.} \\
4 \\
\hline
\end{array}
$$

∴ of the ball $= 113{\cdot}0976$ sq. feet.

$$
\begin{array}{r}
113{\cdot}0976 \\
144 \\
\hline
4523964 \\
4523904 \\
1130976 \\
\hline
\end{array}
$$

16286·0544 sq. inches

16286·0544 sq. inches.
4*d*. per sq. inch.

Cost = 65144·2176 pence.
12)————
2,0)542,8 8*d*.

£271 8 8

31. Determine the content of the lesser segment of a prolate spheroid, cut off by a plane drawn perpendicular to the major axis through the middle point between the centre and extremity of the axis: the two axes of the generating ellipse being 12 and 8 feet respectively.

$$\frac{(\text{revolv. axis})^2}{(\text{fixed axis})^2} = \tfrac{64}{144} = \tfrac{4}{9}.$$

3 times fixed axis = 36
2 height of seg. = 6

30
4

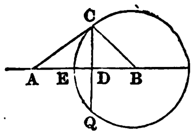
9)120

= 3^2 = (height)

97
236

[illegible]

32. How high above the surface of the earth must a person ascend in order to see one-fourth part of its surface?



Since the surface of any part C E Q of the earth (supposed to be a sphere) varies as E D; therefore if the radius E B be bisected in D, and D C be drawn perpendicular to E B, a tangent C A drawn from C will meet the diameter of the sphere produced in A, the point in which the person must be placed so that he may see one-fourth of the whole surface of the earth.

$$\text{Now by sine }\Delta\text{s. } AB : BC :: BC : BD.$$
$$\text{or } AB : BC :: BC : \tfrac{BC}{2}$$
$$\therefore AB = \frac{2\,BC^2}{BC} = 2\,BC.$$
$$\text{And } AE = BC = \text{the earth's radius.}$$

Hence a person elevated above the earth's surface to a distance equal to the earth's radius, may thence view one-fourth of the whole surface of the earth.

In a similar way it must be shown that in order to see one-third of the surface, A E must = diameter of sphere.

33. Find the content of a Winchester bushel of mustard seed, heaped up in the form of a cone to the height of 6 inches above the top of the bushel; the internal diameter being $18\frac{1}{2}$ inches, and its depth 8 inches.

$$
\begin{array}{r}
\cdot 7854 \\
342\cdot 25 = (18\cdot 5)^2 \\
\hline
136900 \\
171125 \\
273800 \\
239575 \\
\hline
268\cdot 803150 = \text{area of cone's base.}
\end{array}
$$

$$
\begin{array}{rl}
268{\cdot}803150 & = \text{area of cone's base.} \\
8 & \\
\hline
2150{\cdot}425200 & = \text{content of cylindrical part.} \\
537{\cdot}606300 & = \text{content of conical part.} \\
\hline
\text{Content} = 2688{\cdot}0315 & \text{cubic inches.}
\end{array}
$$

34. A person wants a cylindrical vessel 3 feet deep, that shall contain twice as much as another cylindrical vessel whose diameter is $3\frac{1}{2}$ feet, and altitude 5 feet. Find the diameter of the required vessel.

Let x = the diameter of the required cylinder's base.

$$x^2 \times {\cdot}7853 \times 3 = 2 \times (3{\cdot}5)^2 \times {\cdot}7854 \times 5.$$
$$3\,x^2 = 10 \times (3{\cdot}5)^2 = 122{\cdot}5.$$
$$x^2 = 40{\cdot}8333.$$
$$\therefore x = 6{\cdot}39 \text{ feet.}$$

35. Two persons agreed to drink off a quart of beer. Now the first person drank till the surface of the liquor touched the opposite edge of the bottom; he then gave the remainder to his companion. Which of these persons drank the larger share; and supposing the beer to cost 6*d.*, what ought each person to pay? the pot being the frustum of a cone whose end diameters are 3 and 4 inches respectively, and its depth 5 inches.

Product of the two diameters = 12 (see Prob. XXIII. in Solids.)

$$
\begin{array}{rl}
\text{and } \sqrt{12} = & 3{\cdot}4641 \\
& 3 = \text{lesser diameter.} \\
\hline
& 10{\cdot}3923 \\
& 16{\cdot} \quad = \text{(greater diameter).} \\
\hline
4 - 3 = 1) & 5{\cdot}6077 \\
\hline
& 5{\cdot}6077
\end{array}
$$

```
        5·6077
            20 = 4 × 5
      ----------
      112·1540
        ·2618
      ----------
        897232
       112154
      672924
     224308
```

Content of the liquor given to the comp. $\Big\}$ = 29·3519172 cubic inches.

Also content of whole pot $= \dfrac{4^3 - 3^3}{4 - 3} \times \cdot 7854 \times \frac{5}{3}$ cub. inch.

(Prop. IX. Appendix.)

$= 37 \times \cdot 7854 \times \frac{5}{3} = 48\cdot 433$ cub.in.

Hence as 48·433 : 6 pence : 29·3519172

```
                              6
                     ----------
48·433)176·1115032(3.6361
       145299
       ------
        308125
        290598
        ------
         175270
         145299
         ------
          299713
          290598
          ------
            91152
          ------
```

Hence the person who drinks first must pay 2·3639 pence, and the person who drinks next must pay 3·6361 pence; *i. e.* their shares of the payment are $2\frac{1}{4}d. + \cdot 4556$ q., $3\frac{1}{2}d. + \cdot 5444$ q.

36. Three persons having bought a conical sugar loaf wish to divide it into three equal parts by sections parallel

the base; it is required to find the altitude of each person's share; the altitude of the loaf being 20 inches.

Now since similar solids are as the cubes of their homologous sides, therefore, if $x =$ altitude of highest portion, and $y =$ that of the middle portion.

$$\text{Whole cone : upper part} :: 20^3 : x^3$$
$$3 : 1 :: 20^3 : x^3$$
$$\therefore x + y = \frac{20}{\sqrt[3]{3}} = 13{\cdot}867.$$

Again, whole cone : two upper portions $:: 20^3 : (x+y)^3$

$$\text{Or, } 3 : 2 :: 20^3 : (x+y)^3$$
$$\therefore x + y = 20\,\frac{\sqrt[3]{2}}{\sqrt[3]{3}}$$
$$\therefore = 20.\frac{\sqrt[3]{2}}{\sqrt[3]{3}} - 20.\frac{1}{\sqrt[3]{3}} = 20.\frac{\sqrt[3]{2}-1}{\sqrt[3]{3}} = 3{\cdot}604.$$

Consequently $20 - \{3{\cdot}604 + 13{\cdot}867\} = 20 - 17.471 = 2{\cdot}529 =$ altitude of the lowest part of the loaf.

37. A solid in the form of a paraboloid is to be divided into three equal parts by planes parallel to the base. Determine the portion of the axis intercepted by the planes; supposing the height of the solid to be 25 inches.

Let $x =$ altitude of the highest part,
and $y =$ altitude of middle part.

$$\text{Whole solid : highest part} :: 25^2 : x^2$$
$$3 : 1 :: 25^2 : x^2$$
$$\therefore x = 25\,.\,\frac{1}{\sqrt{3}} = 14{\cdot}437.$$

Again, whole solid : two upper portions $:: 25^2 : (x+y)^2$.

$$\text{Or, } 3 : 2 :: 25^2 : (x+y)^2$$
$$\therefore x + y = 25\,.\,\frac{\sqrt{2}}{\sqrt{3}}$$
$$\text{and } y = 25\,.\,\frac{\sqrt{2}-1}{\sqrt{3}} = 5{\cdot}977$$

Consequently the altitude of the lowest portion $=$ 4·58 ...es.

38. A gentleman having a rectangular bowling green, 300 feet long, and 200 feet broad, wishes to raise its surface one foot, by means of the earth to be dug out of a ditch that surrounds it: to what depth must the ditch be dug, supposing its breadth to be everywhere 8 feet?

Length = 300 feet.
Breadth = 200 feet.

60000
1 foot deep.

60000 cubic feet to be dug out of the ditch.

Let x = depth required.

Then the content of the earth dug out $= 2 \times 316 \times 8 \times x + 2 \times 200 \times 8 \times x.$

$= 5056\,x + 3200\,x = 8256\,x.$

$\therefore 8256\,x = 60000$

$x = \frac{60000}{8256} = 7\frac{23}{86}$ feet, as required.

39. A cubical foot of copper is to be drawn into wire, the diameter of which is $\frac{1}{40}$ of an inch. Find the length of the wire.

Let x = the length of the wire in inches.

Then since the diameter of the wire = ·025 inch, the content of the wire $= \cdot 7854 \times (\cdot 025)^2 \times x$ cubic inches.

```
   ·025
   ·025
   ----
    125
    50
 -------
 ·000625
   ·7854
 -------
    2500
   3125
  5000
 4375
-----------
·0004908750
```

$\therefore x = \frac{1728}{\cdot 000490875}$ inches $= 3520244 \cdot 4614$ inches $=$ $97784 \cdot 55$ yards $= 55\frac{1}{2}$ miles nearly; the error being in defect.

40. If a cylindrical cistern be 26·5 inches in diameter, and 52·5 inches in depth: required the diameter of a cylindrical cistern that will hold twice as much as the former, supposing its depth to be the same.

Let $x =$ diameter of the required cylinder.
Then $\cdot 7854 \times x^2 \times 52\cdot 5 =$ its content.

Also $\cdot 7854 \times (26\cdot 5)^2 \times 52\cdot 5 =$ content of the first mentioned vessel.

Hence, from the condition of the problem,
$\cdot 7854 \times x^2 \times 52\cdot 5 = 2 \times \cdot 7854 \times (26\cdot 5)^2 \times 52\cdot 5$
Or $x^2 = 2 \times (26\cdot 5)^2 = 1404\cdot 5$
$\therefore x = \sqrt{1404\cdot 5} = 37\cdot 47$ inches, the diameter required.

41. Find the diameter of a globe whose solidity and superficial content are expressed by the same number.

Let $x =$ the diameter sought.
Then if $\pi = \cdot 7854 =$ area of a circle to diameter unity;
$4 \pi x^2 =$ superficial content of the globe.
And $\frac{2}{3} \pi x^3 =$ content.
Hence, if $\frac{2}{3} \pi x^3 = 4 \pi x^2$
$x = 6 =$ the diameter sought.

42. Determine the value of the frustum of a marble cone at 12 shillings per cubic foot, the diameter of the greater end being 4 feet, that of the lesser end $1\frac{1}{2}$ feet, and the slant side 8 feet.

$4^3 = 64$ See Problem IX. in Solids.
$(1\cdot 5)^3 = 3\cdot 375$

Difference $= 60\cdot 625$

```
2·5)60·625(24·25
    50
    ---
    106
    100
    ---
     62
     50
     ---
     125
     125
     ---
```

Also the altitude of the frustum $= \{8^2-(1·25)^2\}^{\frac{1}{2}} =$ $\overline{·4375}$ $= 7·901$.

```
Hence 24·25
       ·7854
      ------
        9700
      12125
     19400
    16975
    --------
    19·045950
       2·633 = 1/3 the altitude.
   ----------
     5713785
    5713785
  11427570
  3809190
  ----------
```

Content = 50·14798635 cubic feet.
12*s*. per cubic foot.

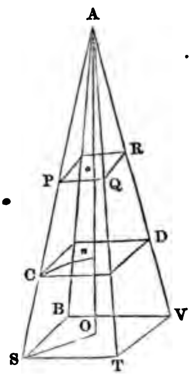
```
 601·77583620
           12
 ------------
   9·31003440
            4
 ------------
   1·24013760
```

Value required = 601*s*. 9¼*d*. = £30 1s. 9¼d.

43. Three persons bought a piece of timber, which shaped like the frustum of a square pyramid. A side of greater end was 3 feet, a side of the lesser end 1 foot, an length of the timber 18 feet: determine the altitude of man's share; all the shares being supposed equal.

$$\begin{aligned} \text{Area of greater end} &= 9 \\ \text{——— lesser end} &= 1 \\ \text{Square root of their product} &= 3 \\ & \overline{13} \\ & 6 = \tfrac{1}{3} \text{ the hei} \\ \text{Content} &= \overline{78} \text{ cubic feet.} \end{aligned}$$

Let x = altitude of the highest portion;
and y = altitude of the middle portion.



Let P S T V R be the given frustum, whose ends S are squares. Produce S P, V R till they meet in A, the A P R be a pyramid similar to the pyramids A S V, A C D.

Now A O : A o : : A S : A P : : S T : P Q : : 3 :
∴ O o : A o : : 2 : 1, or A $o = \frac{1}{2}$ O o = 9 feet.

And content of pyramid A P R = area of P R $\times \frac{A\,o}{3}$

t.

t similar solids are as the cubes of their homologous
;

$\therefore$ solid ACD : solid APR : : A n^3 : A o^3

29 cubic feet : 3 cubic feet : : $(9 + x)^3 : 9^3$

$$9 + x = 9 \sqrt[3]{\frac{29}{3}} = 19{\cdot}172.$$

ınd $x = 10{\cdot}172 = on =$ altitude of the highest portion.

ıo solid whose altitude is $(19{\cdot}172 + y)$: solid ACD : :
$72 + y)^3 : (19{\cdot}172)^3$

ər $55 : 29 : : (19{\cdot}172 + y)^3 : (19{\cdot}172)^3$

$$\therefore 19{\cdot}172 + y = 19{\cdot}172 \times \sqrt[3]{\frac{55}{29}}$$

Now log. 55 is 1·7403627
log. 29 is 1·4623980

3)0·2779647

·0926549

Log. 19·172 is 1·2826674

23·7313 = 1·3753223

ence $19{\cdot}172 + y = 23{\cdot}7313$

$\therefore y = 23{\cdot}7313 - 19{\cdot}172 = 4{\cdot}5593.$

ısequently the altitude of the lowest frustum = 18 —
13 = 3·2687 feet.

Determine the diameter of a circle whose area is equal e whole surface of a conical frustum, the altitude of is 3 feet, and end diameters 5 and $2\frac{1}{2}$ feet respectively.

3·1416
5

15·7080 = perimeter of the base.
7·8540 = perimeter of the upper end.

23·5620 = sum of perimeters.

o the slant height of the frustum $= \sqrt{3^2 + (1{\cdot}25)^2} =$
$\overline{625} = 3{\cdot}25.$

$$
\begin{array}{r}
23\cdot5620 \\
3\cdot25 = \text{slant height.} \\
\hline
1178100 \\
47124 \\
70686 \\
\hline
2)76\cdot576500 \\
\hline
\end{array}
$$

$\therefore$ surface of frustum $= 38.28825$ square feet. Prob. VI
Area of lesser end $= 4\cdot90875$ [Appen. to Mensuration
Area of greater do. $= 19.6350$

$62\cdot83200 =$ *whole* surface of the frustum.

Hence if $x =$ diameter of the required circle,

$x^2 \times \cdot7854 = 62\cdot832$

and $x^2 = \dfrac{62\cdot832}{\cdot7854} = 80$

$\therefore x = 8\cdot944$ feet the diameter sought.

45. Determine the bore of a cannon which is cast for a lb. ball; so that the allowance for windage may be $\frac{1}{10}$ of inch: supposing an iron ball of 4 inches diameter to weig lbs.

Here $9 : 4^3 :: 24 : \delta^3$

$\therefore \delta = 4 \dfrac{\sqrt[3]{24}}{9} = 4 \dfrac{\sqrt[3]{8}}{3}.$

$\text{Log } \delta = \log 4 + \frac{1}{3}\{\log 8 - \log 3\}$
$= \cdot6020600 + \frac{1}{3}\{\cdot9030900 - \cdot5771213\}$
$= \cdot6020600 + \cdot1419896 = \cdot7440496$

$\therefore \delta = 5.5468 =$ diameter of the ball; to which if we a $\frac{1}{10}$ for the space between the ball and the inner sides of the g we shall have $5\cdot6468$ inches the bore required.

46. How many shot are there in a complete oblong pile ngth of whose base contains 48, and the breadth 30 sh

$$
\begin{array}{r l}
48 & = m \\
3 & \\
\hline
144 & = 3m \\
1 & \\
\hline
145 & = 3m + 1 \\
30 & = n \\
\hline
115 & = 3m - n + 1 \\
30 & \\
\hline
3450 & \\
31 & \\
\hline
6)106950 & \\
\hline
\end{array}
$$

quired = 17825 balls.

w many shot are there in an unfinished oblong pile, same length and breadth as in the last problem; the breadth of the highest course being 24 and 6?

d the number of shot contained in a complete pile, th and breadth are 23 and 5 respectively, then the between this number and 17825, as found in the last ill give the number sought.

$$
\begin{array}{r}
23 \\
3 \\
\hline
69 \\
1 \\
\hline
70 \\
5 \\
\hline
65 \\
5 \\
\hline
325 \\
6 \\
\hline
6)1950 \\
\hline
325 \\
\hline
\end{array}
\qquad
\begin{array}{r}
17825 \\
325 \\
\hline
\text{No.} = 17500 \text{ shot.}
\end{array}
$$

48. Determine the number of shot in a triangul side of whose base contains 50 shot.

```
     50
     51
   ----
   2550
     52
   ----
   5100
  1275
  -----
6)132600
  ------
```

No. req. = 22100 shot.

49. How many shot are there in an unfinished of 12 courses; the length and breadth of the top 40 and 10 shot respectively?

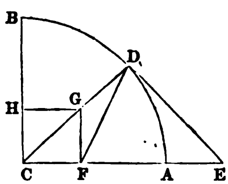
Since there are 12 courses or layers of balls, a length and breadth of the base decrease by one b course, therefore the length and breadth of the b 51 and 21 shot respectively.

```
    51
     3
   ---
   153
     1
   ---
   154
    21
   ---
   133
    21
   ---
   133
  266
  ----
  2793
    22
  ----
  5586
 5586
 -----
6)61446
  -----
  10241
```

10241 = No. of balls in the pile wher

```
   39
    3
 ----
  117
    1
 ----
  118
    9
 ----
  109
    9
 ----
  981
   10
 ----
6(9810
 ----
 1635
10241
 -----
 8606 = No. of balls required.
 -----
```

circle is inscribed in the quadrant of a given circle; .e the difference between the area of the quadrant and of the inscribed circle: supposing the radius of the ; to be 5 feet in length.



B C be the given quadrant; bisect the angle *A C B* by ;ht line *C D*: and at *D* draw *D E*, touching the quad

rant, and meeting C A produced in E. Make C F = ı
From F draw F G, at right angles to A C; then is G the ce
of the circle inscribed in the quadrant A D B. For since
= A E, ∴ C A = F E; but C A = C D = D E, because
angle D C E = angle D E C = $\frac{1}{2}$ a right angle; hence D E =
and angle E D F = angle E F D; consequently the angle G
= angle G F D, and G D is therefore equal to G F. If G ı
drawn perpendicular to C B, it is evident that G F = G H;
therefore a circle described about centre G at the distance
will pass through H and F, touch the lines C B, C A, an
inscribed in the quadrant B C A.

Let C D $= r$, and $\pi = 3\cdot1416$ = area of a cir. to radius u

Then C E $= r\sqrt{2}$.

And C F $= r\,\{\sqrt{2}-1\}$ = radius of the inscribed circ

∴ C F$^2 = r^2 . \{3 - 2\sqrt{2}\}$.

And the area of inscribed circle $= \pi r^2\,\{3 - 2\sqrt{2}\}$.

Also the area of the quadrant A D B C $= \frac{\pi r^2}{4}$.

$$\therefore \text{ the difference } = \frac{\pi r^2}{4} - \pi r^2\{3 - 2\sqrt{2}\}.$$

$$= \pi r^2\,\{\tfrac{1}{4} - 3 + 2\sqrt{2}\} = \frac{\pi r^2}{4}\,\{8\sqrt{2} - 11\} = 6\cdot$$

square feet.

51. An irregular lump of ore weighs 15 ounces avoir
in air, and 13 ounces in water; another mass weig
ounces in air, and 14 in water; determine the rates of
respective densities?

Let Δ, Δ' be their respective densities,

and s = specific gravity of water.

Then weight lost in water : absolute weight : : s :

$$\text{or } 2 : 15 :: s : \Delta = \frac{15}{2}.s.$$

Again,

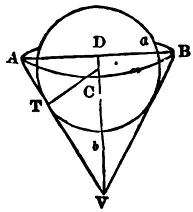
$$\therefore 3 : 17 :: s : \Delta' = \frac{17}{3}.s.$$

$$\therefore \Delta : \Delta' :: \frac{15}{2}.s : \frac{17}{3}.s :: 45 : 34, \text{ the required r}$$

their densities, or specific gravities.

52. A solid sphere of iron, whose diameter is 4 in
ly let fall into a conical glass full of wine; it is

letermine how much wine will run over; the altitude of the sel being 6 inches, and the diameter or width of the glass iches.



Let A V B be the conical glass, into which the sphere is ntly let fall. Let C be the centre of the sphere, and draw ' from C, to the point where the sphere touches the side of : glass.

Now $\text{VB} = \sqrt{\text{VD}^2 + \text{DB}^2} = \sqrt{6^2 + (2\cdot5)^2}$
$= \sqrt{36 + 6\cdot25} = \sqrt{42\cdot25} = 6\cdot5 =$ slant side the glass.

And by similar triangles,

$\text{DB} : \text{BV} :: \text{CT} : \text{CV}$,

or $2\cdot5 : 6\cdot5 :: 2 : \text{CV} = \dfrac{13}{2\cdot5} = 5\cdot2 =$ distance of ball's ntre from the vortex V of the glass.

Hence the ball sinks with its centre to the depth ·8 of an inch elow the plane of the base A B; consequently the diameter is nmersed to the depth 2·8 inches. And the number of cubic iches contained in the segment immersed is equal to the uantity of wine which flows out.

Now $\text{D}a = \sqrt{2\cdot8 \times 1\cdot2}$ (Euclid, book vi. p. 8,) $= \sqrt{3\cdot36}$

$\therefore 3\,\text{D}a^2 = 10\cdot08$

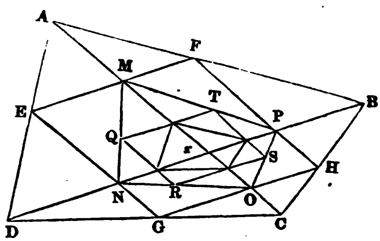
$\text{D}b \;\; = \;\; 7\cdot84$

Sum $= 17\cdot92$

$$
\begin{array}{rl}
\text{Sum} = & 17{\cdot}92 \\
& 2{\cdot}8 = \text{height.} \\
\hline
& 14336 \\
& 3584 \\
\hline
& 50{\cdot}176 \\
& {\cdot}5236 \\
\hline
& 26{\cdot}2721536
\end{array}
$$

26·2721536 cubic inches, $= \frac{3}{4}$ of a pint nearly, imperial measure; the gallon containing 277·274 cubic inches, and the pint 34·6592 cubic inches.

53. If the four sides of any quadrilateral be bisected, the lines joining these points will form a parallelogram. Again, if the diagonals of the quadrilateral be drawn, they will cut the sides of the parallelogram; and by joining the points of section, a figure will be formed similar to the first quadrilateral. Let this process be continually repeated; then if A = the area o the given four-sided figure; determine the sum of the areas of all the quadrilaterals, and of all the parallelograms.



Let A B C D be the proposed quadrilateral. Bisect its sides in the points E F H G. Join F E, E G, G H, H F, and draw the diagonals A C, D B. Now because A F = F B, and A E = E D, ∴ A F : F B : : A E : E D, consequently E F is parallel to D B; Euclid, book vi. p. 2. For the same reason, G H is parallel to D B, and therefore G H and E F being both parallel to D B, are parallel to one another. In the same way it may be shown that E G and F H are each parallel to A C, and therefore the figure E G H F is a parallelogram.

Next, suppose the diagonals A C, D B, to cut the sides of the parallelogram E G H F in the points M, N, O, P. Join M N, N O, O P, P M; and let x be the point in which the diagonals cut one another.

Then since D N : N x : : D G : G C.

And C O : O x : : C G : G D.

∴ the sides D x, C x of the triangle D x C are cut proportionally in the points N, O, hence N O is parallel to D C. In the same manner, it may be shown that N M is parallel to D A, M P to A B, and P O to B C, whence the figure M N O P is similar to A D C B. Again, if the sides M P, P O, O N, N M be bisected in T, S, R, Q, and Q T, T S, S R, R Q be drawn, the figure Q T S R is a parallelogram, similar to the parallelogram E F H G. And if the points in which the diagonals A C, D B, cut the sides of the parallelogram Q R S T be joined, a figure will be formed similar to M N O P, and to A D C B; and the middle points of this last mentioned figure being joined, will form a parallelogram similar to the preceding parallelograms which have been formed.

Now $\triangle$ A E F : $\triangle$ A D B : : A E^2 : A D^2 : : 1 : 4,
and $\triangle$ E D G : $\triangle$ A B C : : D G^2 : D C^2 : : 1 : 4,
and $\triangle$ C G H : $\triangle$ D B C : : C H^2 : C B^2 : : 1 : 4,
and $\triangle$ B F H : $\triangle$ B A C : : B F^2 : B A^2 : : 1 : 4.
∴ The four $\triangle$s A E F, E D G, C G H, B F H : whole quad. : : 1 : 2.

Hence the parallelogram E G H F $= \frac{A}{2}$.

Now the $\triangle$ N M P $= \frac{1}{2}$ parallelogram E N P F,
and $\triangle$ N O P $= \frac{1}{2}$ parallelogram G N P H.
∴ the quadrilateral M N O P $= \frac{1}{2}$ parallelogram E G H F
$= \frac{A}{4}$.

From this construction it appears that there will be a series of quadrilaterals, whose areas are A, $\frac{A}{4}$, $\frac{A}{4^2}$, $\frac{A}{4^3}$, &c., ad infinitum; and a series of parallelograms whose areas are $\frac{A}{2}$, $\frac{A}{8}$, $\frac{A}{32}$, &c., *ad infinitum.*

Now $A + \frac{A}{4} + \frac{A}{4^2} +$ &c., ad infinitum $= \frac{A}{1-\frac{1}{4}} = \frac{4A}{3}$.

And $\frac{A}{2} + \frac{A}{8} + \frac{A}{32} +$ &c., ad infinitum, $= \frac{\frac{1}{2}A}{1-\frac{1}{4}} = \frac{2A}{3}$.

54. A sphere, whose diameter is 6 feet, is cut by two parallel planes distant 2 feet 6 inches from each other: find the convex surface of the frustum intersected by the planes.

Now the convex surface of any zone or segment is found by multiplying its altitude by the circumference of the sphere; hence

$$\begin{array}{rl} 3{\cdot}1416 & \\ 6 & \\ \hline 18{\cdot}8496 & = \text{circumference of sphere.} \\ 2{\cdot}5 & = \text{altitude of frustum or zone.} \\ \hline 942480 & \\ 376992 & \\ \hline \end{array}$$

Surface $= 47{\cdot}12400$ square feet.

55. If the linear side of a certain cube be increased one inch, the surface of the cube will be increased 246 square inches. Determine the side of the cube.

Let $x =$ a side of the cube.

Then $6\,x^2 =$ whole surface of the cube.

But if the side be increased by 1 inch,

$6\,(x + 1)^2 =$ whole surface of the cube.

Hence $6\,\{(x + 1)^2 - x^2\} = 246$.

Or $(x + 1)^2 - x^2 = 41$.

$\therefore 2\,x + 1 = 41$.

And $2\,x = 40$.

i. e. $x = 20$ inches, the side required.

56. If the linear side of a certain cube be increased one inch the solidity of the cube will be increased 91 inches. Determine the side of the cube.

Let $x =$ a side of the cube.

Then $x^3 =$ content of the cube.

Hence $(x + 1)^3 - x^3 = 91$ by the question.

Or $3x^2 + 3x + 1 = 91$.

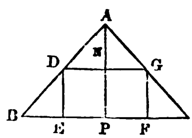
$\therefore x^2 + x = 30$.

And $x^2 + x + \frac{1}{4} = \frac{121}{4}$.

$\therefore x + \frac{1}{2} = \frac{11}{2}$.

And $x = \frac{11-1}{2} = \frac{10}{2} = 5$ inches, the length of the side required.

57. Express the side of a square inscribed in a triangle, in terms of the three sides a, b, c; supposing b to be the base.



Let A B C be the triangle, whose sides A B, B C, C A, are represented by a, b, c, respectively. And suppose D E F G to be a square inscribed in the triangle; also let A P be perpendicular to the base B C; and x = a side of the square.

Then by similar triangles,

A P : B C : : A N : D G.

Or A P : b : : A N : x.

$\therefore$ A P : N P : : $b : b - x$.

Hence A P $= \frac{bx}{b-x}$.

And $\frac{\text{AP . BC}}{2} = \frac{b^2 x}{2(b-x)}$

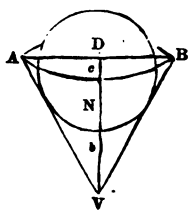
i. e. area of $\triangle$ A B C $= \frac{b^2 x}{2(b-x)}$

But area of the triangle $= \{ \frac{1}{2} p (\frac{1}{2} p - a) . (\frac{1}{2} p - b) . (\frac{1}{2} p - c) \}^{\frac{1}{2}}$

$$\therefore \frac{b^2 x}{2(b-x)} = \{\tfrac{1}{2}p\,(\tfrac{1}{2}p\ a) \cdot (\tfrac{1}{2}p - b) \cdot (\tfrac{1}{2}p - c)\}^{\frac{1}{2}}$$

$$\therefore x = 2\,b.\ \frac{\{\tfrac{1}{2}p\,(\tfrac{1}{2}p - a) \cdot (\tfrac{1}{2}p - b) \cdot (\tfrac{1}{2}p - c)\}^{\frac{1}{2}}}{b^2 + 2\{\tfrac{1}{2}p(\tfrac{1}{2}p - a) \cdot \tfrac{1}{2}p - b) \cdot (\tfrac{1}{2}p - c)\}^{\frac{1}{2}}}$$

58. If the dimensions of the sphere and cone be the same as in Problem LII., and the cone only half full; determine the length of the sphere's axis immersed in the wine.



Let N be that point in the cone's axis, through which, if a plane pass parallel to A B, the cone will be divided into two equal parts.

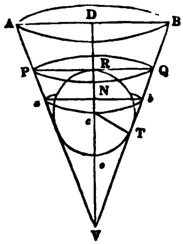
Content of whole cone : $\frac{1}{2}$ content : : V D^3 : V N^3.

$$\text{Or } 2 : 1 :: 6^3 : \text{V N}^3 = \frac{6^3}{2} = 108.$$

$\therefore$ V N $= \sqrt[3]{4} = 4 \cdot 7622$ inches $=$ the altitude at which the wine stands in the glass when half full. Also in Problem LII., it was demonstrated that D$b = 2 \cdot 8$ inches ; $\therefore$ V$b = 3$ consequently V N $-$ V $b = 4 \cdot 7622 - 3 \cdot 2 = 1 \cdot 5622$ inches that part of the sphere's diameter which is immersed, when glass is half full of wine.*

59. The cone being the same as before, and only half find the diameter of that sphere which being gently let into the glass, will be just covered by the fluid.

* N is the point where the fluid stands before the sphere is put in after the sphere is put in, the surface rises a distance x, such, that the co of the conic frustum, whose altitude is x, is equal to the content of a seg of a sphere whose altitude $= 1 \cdot 5622$ to radius 2. This altitude = is four ; inch ; therefore the whole axis immersed $= 2 \cdot 3437$ inches.



Let $a\ b$ be the surface of the wine before the sphere is put and when the sphere is immersed, suppose the surface to e to P Q, which touches the sphere in R. Then it is evident t the content of the frustum P $a\ b$ Q — content of the ierical segment whose height is N R = content of the sphe-al segment whose height is N o : *i. e.* the frustum P $a\ b$ Q= sphere R o.

Let x = radius of the required sphere in $\pi = \cdot7854$; and content of the sphere $= 8\,x^3 \times \frac{2\,\pi}{3} = \frac{16\,\pi\,x^3}{3}$.

Then the glass being half full, the fluid stands at the altitude N = 4·7622 inches.

And V D : D B : : V N : N b.

Or 6 : 2·5 : : 4·7622 : N b = 1·9843 inches.

And V D : V B : : V N : V b.

Or 6 : 6·5 : : 4·7622 : : V b = 5·15905 inches.

For convenience let N V = a, N b = b, V b = c.

Then $c : b :: $ C V : x ∴ C V $= \frac{c\,x}{b}$.

And V R $= x + c\,x = \frac{b+c}{b} \cdot x$.

Now cone V P Q : cone V $a\ b :: \left(\frac{b+c}{b}\right)^3 \cdot x^3 : a^3$

And frust. P $a\ b$ Q : cone V $a\ b :: \left(\frac{b+c}{b}\right) \cdot x^3 - a^3 : a^3$

$$\frac{16\,\pi\,x^3}{3} : \frac{4\,\pi\,b^2\,a}{3} :: \left(\frac{b+c}{b}\right)^3 \cdot x^3 - a^3 : a^3$$

$$4\,x^3 : b^2 a :: \left(\frac{b+c}{b}\right)^3 x^3 - a^3 : a^3$$

Whence $x = \dfrac{a\,b}{\sqrt[3]{(b+c)^3 - 4\,a^2\,b}} = 1\cdot66$ inches.

If the same were required, on the supposition that the cone be $\frac{1}{3}$ full of wine, the diameter will be found $= 2\cdot446$ inches nearly.

60. Find the dimensions of a hollow conical frustum that will hold 13 gallons, the end diameters being as 5 to 3, and the depth of the vessel 12 inches.

Now $5^3 - 3^3 = 98$

$5 - 3 \;\;= 2$

$\therefore \dfrac{5^3 - 3^3}{5 - 3} = \frac{98}{2} = 49$

$\qquad\qquad\qquad\quad\;\; 4 = \frac{1}{3}$ the alt.

$\qquad\qquad\qquad\; 196$

Now 1 gall. imp. meas. contains 277·274 cubic inches.

```
                              13
                          ------
                          831822
                         277274
                         -------
           ·7854)3604·562(4589·46
                 31416
                 -----
                  46296
                  39270
                  -----
                   70262
                   62832
                   -----
                    74300
                    70686
                    -----
                     36140
                     31416
                     -----
                      47240
                      47124
                      -----
                        116
```

And $\dfrac{4589{\cdot}46}{196} = 23{\cdot}415612$

$\therefore \sqrt{23{\cdot}415612} = 4{\cdot}839$, nearly.

Hence $4{\cdot}839 \times 3 = 14{\cdot}517 =$ the bottom diameter.

and $4{\cdot}839 \times 5 = 24{\cdot}195 =$ the top diameter.

Otherwise.

Let $x =$ the top diameter.

Then bottom diameter $: x :: 5 : 3$

And bottom diameter $= \dfrac{5\,x}{3}$.

And $\dfrac{\left(\dfrac{5\,x}{3}\right)^3 - x^3}{\dfrac{5\,x}{3} - x} \times 4 \times {\cdot}7854 =$ content in cubic inches.

This quantity reduced becomes $= 17{\cdot}1043\,x^2$ cubic inches.

But 13 gallons imp. measure $= 3604{\cdot}562$ cubic inches.

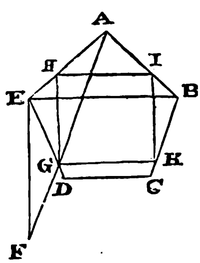
$\therefore 17{\cdot}1043\,x^2 = 3604{\cdot}562$

and $x^2 = \dfrac{3604{\cdot}562}{17{\cdot}1043} = 210{\cdot}7401$

$\therefore x = \sqrt{210{\cdot}7401} = 14{\cdot}516$ inches $=$ the top diameter

And $\dfrac{5\,x}{3} = 24{\cdot}193$ inches $=$ bottom diameter.

61. Determine the side of a square inscribed in a regular pentagon whose side is 5 feet in length.



We will first inscribe a square in the given pentagon A B C D E, one of whose equal sides is 5 feet in length. Join E B; and

draw E F perpendicular and equal to E B. Join A F, cutting the side E D in G. Draw G H parallel to E F, H I parallel to E B, I K parallel to E F, and join K G: then H I K G will be a square, a side of which is sought in terms of A E.

Since H G is parallel to E F, and H I to E B,

H G : E F : : A H : A E : : H I : E B,

but E F = E B, ∴ H G = H I; and since A E = A B,

Therefore H E = I B; also G K, and D C being parallel to E B and D E = B C, therefore E G = B K. Hence H E, E G = I B, B K each to each, and the angle H E G = the angle I B K: ∴ H G = I K, and the angle E H G = B I K, and H G, I K are parallel; and they have been proved to be equal, ∴ G K equal and parallel to H I; and the figure H I K G is equilateral; but it is also rectangular, because the angle at H is a right angle; consequently the figure is a square.

Now in order to find a value of H G, we have the angle E = 108°, Eucl. B, 1. Prop. 32. Cor. I. Also the angle A E B equal to the angle A B E = 36°; hence

$$\text{A E : E B : : sin. } 36^{\circ} : \text{sin. } 72^{\circ}$$
$$: : \text{sin. } 36^{\circ} : 2 \text{ sin. } 36 \text{ cos. } 36$$
$$: : 1 : 2 \text{ cos. } 36^{\circ} : : 1 : \tfrac{1}{2}(1 + \sqrt{5})$$

$$\therefore \text{E B} = \tfrac{5}{2}(1 + \sqrt{5}). \quad \text{Put } x = \text{G H}.$$

Then A E : E B : : A H : H I

$$\text{or } 1 : \tfrac{1}{2}(1 + \sqrt{5}) : : \text{A H} : x$$

$$\therefore \text{A H} = \frac{2x}{1 + \sqrt{5}}, \text{ and E H} = 5 - \frac{2x}{1 + \sqrt{5}}.$$

Also E H : H G : : sin. 18° : sin. 108°

$$: : \text{sin. } 18^{\circ} : \text{sin. } 72^{\circ}$$
$$: : \tfrac{1}{4}(\sqrt{5} - 1) : \tfrac{1}{4}\sqrt{10 + 2\sqrt{5}}$$

$$\text{Or } 5 - \frac{2x}{1 + \sqrt{5}} : x : : \sqrt{5} - 1 : \sqrt{10 + 2\sqrt{5}}$$

$$\therefore x(\sqrt{5} - 1) = 5\sqrt{10 + 2\sqrt{5}} - 2x \cdot \frac{\sqrt{10 + \ldots}}{1 + \ldots}$$

$$\text{Or } 4x = 5\sqrt{(10+2\sqrt{5})\cdot(6+2\sqrt{5}} - 2x\sqrt{10+2\sqrt{5}}$$
$$= 5\sqrt{80+32\sqrt{5}} - 2x\sqrt{10+2\sqrt{5}}$$
$$\therefore 2x = 5\sqrt{20+8\sqrt{5}} - x\sqrt{10+2\sqrt{5}}$$
$$\therefore x\{2+\sqrt{10+2\sqrt{5}}\} = 10\sqrt{5+2\sqrt{5}}$$
$$\text{And } x = 10\cdot\frac{(5+2\sqrt{5})^{\frac{1}{2}}}{2+(10+2\sqrt{5})^{\frac{1}{2}}} = 5\cdot3025 \text{ feet} = \text{the}$$
side of the inscribed square.

62. Take a cubic inch of gold, and beat it into a circular plate whose thickness is $\frac{1}{50}$ of an inch. Find the diameter of the circle.

Let x = the required diameter.

The circular plate will in fact be a cylinder, whose content is 1 cubic inch, altitude ·02 of an inch, and base diameter $= x$, in inches.

$$\therefore x^2 \times \cdot7854 \times \cdot02 = 1$$
$$\text{or } x^2 \times \cdot015708 = 1$$
$$\therefore x^2 = \frac{1}{\cdot015708} = 63\cdot02521$$
$$\text{And } x = \sqrt{63\cdot02521} = 7\cdot939 \text{ inches, the answer.}$$

63. From a rectangular deal board whose length is 60 inches and breadth 30, I wish to cut off a part whose superficies contains a square foot, by a line parallel to the shorter edge. I then wish to cut off the like quantity by a line parallel to the longer side: and so on. What will be the dimensions of the piece which remains after all these square feet are taken away?

Now the first piece which is cut off is a rectangle whose breadth is 30 inches, and content $= 144$ sq. inches; hence the length $= \frac{144}{30} = 4\cdot8$ inches. And the dimensions of the remaining piece are 30, 55·2. In the next place we have to cut off 144 sq. inches by a line parallel to the longer side; hence the breadth $= \frac{144}{55\cdot2} = 2\cdot6086$. The dimensions of the remaining piece are 55·2, 27·3914.

The length of the next portion cut off $= \frac{144}{27\cdot3914} = 5\cdot293$

The dimensions of the remaining part $= 49{\cdot}907,\ 27{\cdot}3914$

The breadth of the next portion cut off $= \dfrac{144}{49{\cdot}907} = 2{\cdot}8853$

The dimensions of the remaining part are $49{\cdot}907,\ 24{\cdot}5061$

The length of the next portion cut off $= \dfrac{144}{24{\cdot}5061} = 5{\cdot}876$

The dimensions of the remaining part are $44{\cdot}031,\ 24{\cdot}5061$

The breadth of the next part cut off $= \dfrac{144}{44{\cdot}031} = 3{\cdot}2704$

The dimensions of the remaining part are $44{\cdot}031,\ 21{\cdot}2357$

The length of the next part cut off $= \dfrac{144}{21{\cdot}2357} = 6{\cdot}781$

The dimensions of the remaining part are $21{\cdot}2357,\ 37{\cdot}250$

The breadth of the next part cut off $= \dfrac{144}{37{\cdot}25} = 3{\cdot}8657$

The dimensions of the remaining part are $17{\cdot}37,\ 37{\cdot}25$

The length of the next part cut off $= \dfrac{144}{17{\cdot}37} = 8{\cdot}291$

The dimensions of the remaining part are $17{\cdot}37,\ 28{\cdot}959$

The breadth of the next part cut off $= \dfrac{144}{28{\cdot}959} = 4{\cdot}972$

The dimensions of the remaining part are $12{\cdot}398,\ 28{\cdot}959$

The length of the next part cut off $= \dfrac{144}{12{\cdot}398} = 11{\cdot}614$

The dimensions of the remaining part are $17{\cdot}345,\ 12{\cdot}398$

The breadth of the next part cut off $= \dfrac{144}{17{\cdot}345} = 8{\cdot}3$

The dimensions of the remaining part are $4{\cdot}098,\ 17{\cdot}34$ and these are the required dimensions; which being multipli together form a product less than 144 square inches.— appears from the process that there will be 12 pieces c from the board, and that 72 square inches will remain; al $4{\cdot}098 \times 17{\cdot}345 = 71{\cdot}07981$; the difference between whi quantity and 72 arising from the fewness of the decin places taken in the different quotients.

64. A gentleman riding round a circular ring in a tw -heeled carriage, observes that the off-wheel turns round twi g the time in which the near-wheel turns round o if the wheels are each 4 feet high, and distant 5 fee

ing to the statute, what is the circumference of the circle ribed by the outer wheel?

ie circumf. of each wheel $= 3{\cdot}1416 \times 4 = 12{\cdot}5664$.

Then if $x =$ diameter of outer circle,

$x - 10 =$ diameter of inner circle,

Also $3{\cdot}1416\, x =$ circumf. of outer circle,

and $3{\cdot}1416\, (x-10) =$ circumf. of inner circle.

$\therefore 3{\cdot}1416\, x = 3{\cdot}1416\, (x-10) \times 2$

$\therefore x = (x-10) \times 2$

Whence $x = 20 =$ diameter of outer circle.

nd $3{\cdot}1416 \times 20 = 62{\cdot}832$ feet $=$ the circumference of the e described by the outer wheel.

i. Determine the area of a circle inscribed in a triangle se three sides are 14, 16, and 20 feet.

$$\begin{array}{r} 14 \\ 16 \\ 20 \\ \hline \text{Perimeter} = 50 \\ \tfrac{1}{2}\ \text{Perimeter} = 25 \end{array}$$

$$\text{Area} = \sqrt{25 \times 11 \times 9 \times 5} = 15\sqrt{55} = 111{\cdot}243 \text{ sq. feet.}$$

Also if $x =$ radius of the inscribed circle,

$$\frac{x}{2} \cdot (14+16+20) = \text{area} = 111{\cdot}243$$

$$\text{Or } 25\, x = 111{\cdot}243$$

$$\therefore x = \frac{111{\cdot}243}{25} = 4{\cdot}449$$

$$\begin{array}{rl} 3{\cdot}1416 & = \text{area of a circle to rad. 1.} \\ 19{\cdot}7936 & = (4{\cdot}449)^2 \\ \hline 1187616 & \\ 197936 & \\ 791744 & \\ 197936 & \\ 593808 & \\ \hline \end{array}$$

Area $= 62{\cdot}18357376$ sq. feet.

66. Determine the area of a circle described about a triangle whose sides are 14, 16, and 20 feet.

If a perpendicular be drawn from the vertical angle of the triangle upon the side which is 16 feet in length, and another perpendicular be drawn from the centre of the circumscribed circle upon the side whose length is 12 feet, we shall have by similar triangles,

$$14 : \text{alt. of } \triangle :: \text{rad. of circle } (x) : 10$$

$$\therefore \text{ alt of } \triangle = \frac{140}{x}$$

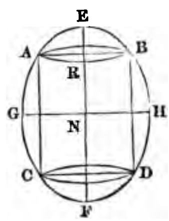
And area of $\triangle = \dfrac{140}{x} \times 8 = \dfrac{1120}{x} = 111 \cdot 243$ as found in last Problem.

$$\therefore x = \frac{1120}{111 \cdot 243} = 10 \cdot 068$$

$$\begin{array}{rl} 3 \cdot 1416 & \text{area of a circle to rad. 1.} \\ 101 \cdot 364 & = (10 \cdot 068)^2 \text{ nearly.} \\ \hline 608184 & \\ 101364 & \\ 405456 & \\ 101364 & \\ 304092 & \\ \hline \end{array}$$

Area $= 318 \cdot 4451424$ sq. feet.

67. Find the content of the greatest cylinder which can be inscribed in a prolate spheroid, whose axes are 12 and 8 feet; the ends of the inscribed solid being perpendicular to the major axis.



Let E A C F D B be the prolate spheroid whose centre is N. ∴ N R $= x$, A R $= y$.

Then $y^2 = \frac{b^2}{a^2}.(a^2 - x^2)$ where a, b are the semi-axes.

Or $AR^2 = \frac{b^2}{a^2}(a^2 - x^2)$

$\therefore$ the content of the cylinder varies as

$\frac{b^2}{a^2}(a^2 - x^2) \times 2x$, or as $(a^2 - x^2) \times x$, which is a maximum.

Hence $a^2\,dx - 3x^2\,dx = 0$.

And $x = \frac{a}{3}$

$\therefore$ alt. of cylinder $= \frac{2a}{3} = AC$

And the content $= \pi\,AR^2\,AC = \frac{\pi b^2}{a^2}\left(a^2 - \frac{a^2}{9}\right) \times \frac{2a}{3}$

$= \frac{16\,\pi\,a\,b^2}{27}$.

Where $\pi = 3\cdot1416$ = area of a circle to radius 1.

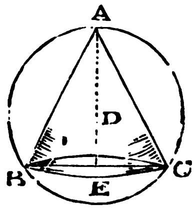
If the value of a, b be substituted in the expression $\frac{16\,\pi\,a\,b^2}{27}$, the actual content of the maximum cylinder $= 178\cdot722$ cubic feet.

68. Find the same as in the last Problem supposing the spheroid to be oblate, and the ends of the inscribed solid are parallel to the major axis.

Here if y = ordinate perpendicular to minor axis, and x = abscissa reckoning from the centre, $y^2 = \frac{a^2}{b^2}(b^2 - x^2)$; and the content of the maximum cylinder will be found $= \frac{16\,\pi\,b\,a^2}{27} =$ 268·083 cubic feet.

NOTE.—If the spheroid be prolate or described round major axis, the sections A C, B D would be ellipses, and the solid A C D B a cylinder with elliptic ends ; and the content of the solid would $\propto$ area of section A C $\times$ A B or $\propto$ product of the two axes of section A C multiplied by A B. By proceeding in this manner we shall find the content of the maximum solid A C D B $= \frac{16\,\pi\,a\,b^2}{3\sqrt{3}}$, the spheroid being prolate, which affords the answer to question 68. If the spheroid be oblate or described round G H, the solid A C B D is a common cylinder with a circular base, and the content of it when a maximum $= \frac{16\,\pi\,b\,a^2}{27}$. Also if the spheroid be oblate, the solid A C D B would have its ends A B, C D, two equal ellipses, and the content of the solid when a maximum $= \frac{16\,\pi\,a^2\,b}{3\sqrt{3}}$.

69. Determine the content of the greatest cone that can be inscribed in a sphere whose radius is 4 feet.



Let D be the centre of the sphere, and suppose D E $= x$, rad. of the sphere $= a$, and $y =$ B E $=$ radius of the cone's base.
Content of cone $\propto$ B E$^2 \times$ A E

$$\propto (a^2 - x^2) \cdot (a + x) \text{ a maximum}$$

$$\therefore -2x dx\,(a + x) + dx\,(a^2 - x^2) = 0$$

$$\text{Or } -2x + a - x = 0$$

$$\therefore x = \frac{a}{3}$$

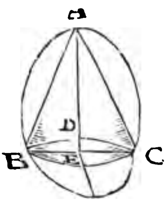
And this being substituted for x in value of $y = (a^2 - x^2)^{\frac{1}{2}}$ gives $y = \dfrac{2\sqrt{2}}{3} =$ radius of cone's base.

Also $\dfrac{4\,a}{3} =$ cone's altitude.

$$\therefore \text{Content} = \pi \times \frac{8}{9}\,a^2 \times \frac{4\,a}{9} = \frac{32\,\pi\,a^3}{81}$$

$= 79{\cdot}432$ cubic feet.

70. Determine the content of the greatest cone that can be inscribed in a prolate spheroid, whose axes are 12 and 8 feet; the base of the inscribed solid being perpendicular to the major axis.



If D E $= x$, and B E $= y$,

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$\therefore$ content of cone $\propto (a^2 - x^2) \cdot (a + x)$

Whence $x = \frac{x}{3} =$ D E

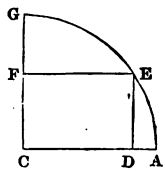
And A E $= \frac{4a}{3}$

Also B E$^2 = \frac{b^2}{a^2}\left(a^2 - \frac{a^2}{9}\right) = \frac{8b^2}{9}$

$\therefore$ Content of cone $= \frac{8\pi b^2}{9} \times \frac{4a}{9} = \frac{32\pi ab^2}{81}$

And if the value of a and b be restored, we shall find the actual content $= 119 \cdot 148$ cubic feet.

71. Find the area of the greatest rectangle that can be inscribed in the quadrant of a circle whose radius is 4 feet.



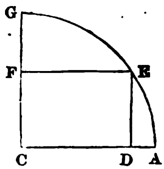
Let C D $= x$, and C A $= a$.
Then D E $= (a^2 - x^2)^{\frac{1}{2}}$
And D E $\times$ C D $= x(a^2 - x^2)^{\frac{1}{2}}$ a maximum.
Or $a^2 x^2 - x^4$ is a maximum.
$\therefore 2a^2 x dx - 4x^3 dx^{\frac{1}{2}} = 0$.

$$\therefore x = \frac{a}{\sqrt{2}} = \cos. 45^\circ.$$

Hence the following construction; bisect the right angle A C G by the straight line C E, and if E D, E F be drawn perpendicular to C A, C G respectively, the rectangle C D E F will be the greatest possible. It also appears that C D E F is a square.—

The area $= x^2 = \frac{a^2}{2} = \frac{16}{2} = 8$ sq. feet.

72. Determine the same for the quadrant of an ellipse wh major and minor axes are 12 and 8 feet.



Let A E G be the quadrant of an ellipse whose centre and semi-axes C A, C G. Let C D $= x$.

Then D E $= y = \frac{a}{b}\,(b^2-x^2)^{\frac{1}{2}}$

$\therefore y\,x = \frac{a}{b}\,x(b^2-x^2)^{\frac{1}{2}} = \square$ C F E D, a maximum.

Hence $b^2\,x^2-x^4$ is a maximum.

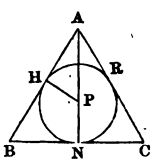
$$\therefore x = \frac{b}{\sqrt{2}}$$

And $y = \frac{a}{b}\left(b^2-\frac{b^2}{2}\right)^{\frac{1}{2}} = \frac{a}{\sqrt{2}}$

$\therefore xy = \frac{ab}{2} =$ area of the maximum rectangle inscrib the quadrant A E G; now $a = 6$ and $b = 4$.

$\therefore xy = \frac{24}{2} = 12$ sq. feet — area required.

73. Find the area of the least isosceles triangle that c described about a circle whose radius is 4 feet.



Suppose A B C to be an isosceles triangle described abo ·rcle H N R, whose radius is 4 feet. Suppose A ·

Then from the nature of the figure,

$AH = (x^2 - 16)^{\frac{1}{2}}$;

And $AH : HP :: AN : NB$

Or $(x^2-16)^{\frac{1}{2}} : 4 :: x+4 : NB = \dfrac{(x+4)\cdot 4}{(x^2-16)^{\frac{1}{2}}}$

And the area of the triangle $ABC \propto AN \cdot NB$

$$\propto \frac{(x+4)^2}{(x^2-16)^{\frac{1}{2}}}$$

ence $2\,dx\,(x+4)\cdot(x^2-16)^{\frac{1}{2}} - \dfrac{x\,dx\,(x+4)^2}{(x^2-16)^{\frac{1}{2}}} = 0$

Or $2(x^2-16) - x(x+4) = 0$

Whence $x = 8 = AP$.

$AN = AP + PN = 12$.

$$NB = \frac{(x+4)\cdot 4}{(x^2-16)^{\frac{1}{2}}} = \frac{48}{\sqrt{48}} = \sqrt{48} = 4\sqrt{3}.$$

And area of $\Delta\, ABC = AN \cdot BB = 48\sqrt{3}$.

It also appears that $AH = 4\sqrt{3}$; or that the triangle ABC equilateral.

74. Find the content of the greatest cylinder that can be in-ribed in a cone whose altitude is 5 feet and base diameter 3 et 6 inches.

Let x = cylinder's altitude.

Then by similar triangles,

$5 : 1{\cdot}75 :: 5-x :$ radius of cylinder's base.

$\therefore$ radius of required cylinder $= {\cdot}35 \times (5-x)$

$\therefore$ content of the cylinder $\propto (5-x)^2 \times x$, a maximum.

Hence $dx\,(5-x)^2 - 2x\,dx\,(5-x) = 0$

Or $5 - x - 2x = 0$

Or $x = \frac{5}{3}$ = cylinder's altitude.

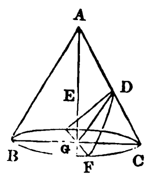
nd radius of cylinder's base $= {\cdot}35 \times \frac{10}{3} = \frac{3.5}{3} = 1{\cdot}166$, &c

Now $(1\cdot1666)^2 = 1.3609$ nearly.
3.1416 = area of cir. to rad. 1.

```
      81654
     13609
    54436
   13609
  40827
  ----------
  4·27540344 = area of cylinder's base.
           5
  ----------
3)21·37701720
  ----------
```

Content = $7\cdot1256724$ cubic feet.

75. Determine the area of the greatest parabola that can be formed by cutting a cone whose base diameter is 2 feet 6 inches, and slant height 4 feet.



Let A B C be the given cone, and E D F the parabola required. Then since the axis D G of the parabola is parallel to A B, the triangles C D G, C A B are similar. For convenience let A B $= a$, C B $= b$: and suppose B G $= x$.

Then C B : B A : : C G : G D.

Or $b : a :: b-x :$ G D$=\frac{a}{b}(b-x)$.

Also from the nature of the circle C G · G B = E G^2

i. e. E G $= (bx-x^2)^{\frac{1}{2}}$. Euclid. B. 6. Prop. 8.

$\therefore$ G D $\times$ G E $\propto (b-x) \cdot (bx-x^2)^{\frac{1}{2}} \propto$ area of parab. E D F.

$\therefore (b-x) \cdot (bx-x^2)^{\frac{1}{2}}$ is a maximum.

ence $-dx\,(bx-x^2)^{\frac{1}{2}}+\dfrac{bdx-2\,xdx}{2(bx-x^2)^{\frac{1}{2}}}(b-x)=0$

$\therefore -2\,(b\,x-x^2)+(b-2\,x)\cdot(b-x)=0$

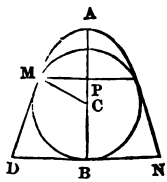
Or $-2x+b-2x=0$

$\therefore x=\dfrac{b}{4}$

And $GD=\dfrac{3\,a}{4}$; $EF=\dfrac{b}{2}\sqrt{3}$

And the area of the maximum parabola $=\frac{2}{3}$ EF·GD $=$ $\times\,\dfrac{3\,a}{4}\times\dfrac{b}{2}\sqrt{3}=\dfrac{a\,b}{4}\sqrt{3}=\frac{10}{4}\sqrt{3}$ square feet $=4\cdot33$ quare feet.

76. Find the area of the least parabola which can circumscribe a circle, whose radius is 3 feet.



Let ADN be the parabola circumscribing the given circle hose centre is C.

Suppose $BC=r$, and $CP=x$.
Then $PM=(r^2-x^2)^{\frac{1}{2}}$

Now PC is the subnormal; and from the nature of the parabola $x=\frac{1}{2}$ principal latus rectum.

Now if $p=$ principal latus rectum,

$p\times AP=MP^2=r^2-x^2$

$\therefore AP=\dfrac{1}{p}(r^2-x^2)$

nd $AB=\dfrac{1}{p}(r^2-x^2)+x+r=\dfrac{1}{2\,x}(r^2-x^2)+x+r=$
$\dfrac{(r+x)^2}{2\,x}$

Also $p \,.\, AB = BD^2$

Or $BD^2 = 2x \,.\, AB = (r + x)^2$

$\therefore BD = r + x$

And $AB \times BD = \dfrac{(r + x)^3}{2x}$ a minimum.

$\therefore 3(r + x)^2 dx \times 2x - 2dx(r + x)^3 -$

Or $6x - 2r - 2x = 0$

$\therefore 4x = 2r$

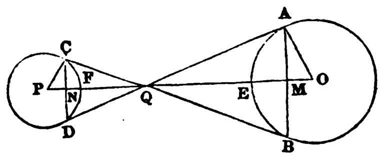
And $x = \dfrac{r}{2} = CP$.

And $AB = \left(\dfrac{3r}{2}\right)^2 \div r = \dfrac{9r}{4}$

$BD = \dfrac{3r}{2}$, and $DN = 3r$.

$\therefore AB \cdot ND \frac{2}{3} \cdot \dfrac{9r}{4} \cdot 3r = \dfrac{9r^2}{2} = \frac{81}{2} = 40\frac{1}{2}$ sq. feet.

77. To find that point in the line joining the centres of given spheres, from which the greatest portion of spherical face is visible.



Let P, O be the centres of the two spheres, and Q the pl the eye. Then tangents drawn from Q to the spheres, wi tercept the parts A E B, C F D of the spherical surfaces will be visible to the eye at Q. Join C D, A B cutting P O and M. Then the surface C F D $\propto$ rad. $\times$ F N; and si A E B $\propto$ rad. $\times$ E M.

Let $PO = \Delta$, $OA = R$, $PC = r$, $PQ = x$.

Then the triangles P C Q, O A Q being right-angled at we have, $QP : PC :: PC : PN$.

Or $x : r :: r : \text{PN} = \dfrac{r^2}{x}$

And $\text{FN} = r - \dfrac{r^2}{x} = \dfrac{rx - r^2}{x}$.

Also $\text{QO} : \text{OA} :: \text{OA} : \text{OM}$.

Or $\Delta - x : \text{R} :: \text{R} : \text{OM} = \dfrac{\text{R}^2}{\Delta - x}$

$\therefore \text{EM} = \text{R} - \dfrac{\text{R}^2}{\Delta - x} = \dfrac{\text{R}(\Delta - x) - \text{R}^2}{\Delta - x}$

ence the whole visible surface

$\propto r^2 - \dfrac{r^3}{x} + \text{R}^2 - \dfrac{\text{R}^3}{\Delta - x}$ a maximum.

$\therefore \dfrac{r^3\, dx}{x^2} - \dfrac{\text{R}^3\, dx}{(\Delta - x)^2} = 0$

Or $\dfrac{r^3}{x^2} = \dfrac{\text{R}^3}{(\Delta - x)^2}$

And $\dfrac{r}{x} = \dfrac{\text{R}^{\frac{3}{2}}}{\Delta - x}$

$\therefore x = \dfrac{r^{\frac{3}{2}}\Delta}{r^{\frac{3}{2}} + \text{R}^{\frac{3}{2}}} = \text{QP}$; which determines the position of

ıe eye.

Now the surface $\text{CFD} = 2\pi\left\{r^2 - \dfrac{r^3}{x}\right\}$

$= 2\pi\left\{r^2 - \dfrac{r^3(r^{\frac{3}{2}} + \text{R}^{\frac{3}{2}})}{r^{\frac{3}{2}}\Delta}\right\} = 2\pi\left\{\dfrac{r^2\Delta - (\text{R}r)^{\frac{3}{2}} - r^3}{\Delta}\right\}$

$=$ visible surface of the lesser sphere.

And the surface $\text{AEB} = 2\pi\left\{\text{R}^2 - \dfrac{\text{R}^3}{\Delta - x}\right\} =$

$2\pi\left\{\text{R}^2 - \dfrac{\quad + \text{R}}{\text{R}\Delta}\right\} = 2\pi\left\{\dfrac{\text{R}^2\Delta - (\text{R}r)^{\frac{3}{2}} - \text{R}^3)}{\Delta}\right\}$

Also,

$\text{CFD} + \text{AEB} = 2\pi\left\{\dfrac{(\text{R}^2 + r^2)\cdot\Delta - 2(\text{R}r)^{\frac{3}{2}} - (\text{R}^3 + r^3)}{\Delta}\right\}$

$= 2\pi\left\{\text{R}^2 + r^2 - \dfrac{(\text{R}^{\frac{3}{2}} + r^{\frac{3}{2}})^2}{\Delta}\right\}$

78. If a conical vessel, whose bottom diameter is 30 inches, be ***nclined to the horizon,*** **so that the surface of the fluid touche**

the upper edge of the base; how many gallons imperial measure are there in the vessel, the upper diameter of the vessel being 19 inches; and the altitude of the frustum 18 inches?

Here the vessel is a conical frustum, whose bottom and top diameters are 30 and 19 inches respectively; and altitude 18 inches.

$$\therefore \text{ content} = \frac{\Delta^2-\delta(\Delta\,\delta)^{\frac{1}{2}}}{\Delta-\delta} \cdot \frac{\pi\,\Delta\,a}{3} \text{ cubic inches.}$$

Here $\Delta = 30$, $\delta = 19$, $a = 18$, $\pi = \cdot 7854$

$\Delta\,\delta = 570$ and $\sqrt{\Delta\delta} = 23{\cdot}8747$

$$\begin{array}{r} 23{\cdot}8747 \\ 19 \\ \hline 2148723 \\ 238747 \\ \hline \end{array}$$

$$\begin{aligned} \delta\sqrt{\Delta\,\delta} &= 453{\cdot}6193 \\ \Delta^2 &= 900 \\ \hline \therefore\ \Delta^2-\delta\sqrt{\Delta\,\delta} &= 446{\cdot}3807 \end{aligned}$$

And $\Delta-\delta = 11$

$$\begin{array}{r} 11)446{\cdot}3807 \\ \hline 40{\cdot}58006 \\ 30 = \Delta \\ \hline 1217{\cdot}40180 \\ 18 = a \\ \hline 97392144 \\ 12174018 \\ \hline 21913{\cdot}2324 \\ \cdot 2618 = \frac{\pi}{3} \\ \hline 1753058592 \\ 219132324 \\ 1314793944 \\ 438264648 \\ \hline \end{array}$$

$277{\cdot}274)5736{\cdot}88424232(20{\cdot}69$ gallons.

r $x : r :: r : PN = \frac{r^2}{x}$

And $FN = r - \frac{r^2}{x} = \frac{rx - r^2}{x}$

lso $QO : OA :: OA : OM$.

r $\Delta - x : R :: R : OM = \frac{R^2}{\Delta - x}$

$EM = R - \frac{R^2}{\Delta - x} = \frac{R(\Delta - x) - R^2}{\Delta - x}$

ce the whole visible surface

$\therefore r^2 - \frac{r^3}{x} + R^2 - \frac{R^3}{\Delta - x}$ a maximum.

$\frac{r^3 dx}{x^2} - \frac{R^3 dx}{(\Delta - x)^2} = 0$

Or $\frac{r^3}{x^2} = \frac{R^3}{(\Delta - x)^2}$

And $\frac{r}{x} = \frac{R^{\frac{3}{2}}}{\Delta - x}$

$\therefore x = \frac{r^{\frac{3}{2}} \Delta}{r^{\frac{3}{2}} + R^{\frac{3}{2}}} = Qr$; which determines the position of eye.

ow the surface $CFD = 2\pi\left\{r^2 - \frac{r^3}{x}\right\}$

$= 2\pi \left\{ r^2 - \frac{r^2(r^{\frac{3}{2}} + \right.$

```
           6394·338
                 19 = δ
         ___________
          57549042
          6394338
        ___________
       11)121492·422
        ___________
          11044·765
              ·2618
        ___________
          88358120
         11044765
        66268590
       22089530
       ____________
277·274)2891·5194770(10·428 galls. answer req
        277274
        ___________
          1187794
          1109096
        _________
            786987
            554548
          ________
           2324397
           2218192
          ________
            106205
```

The vessel when full holds 31·118 gallons, imperial r

80. If a cone be cut by a plane which passes thro edge of the base, the upper part of the cone : whole $\delta^{\frac{3}{2}} : \Delta^{\frac{3}{2}}$; where δ and Δ are the top and bottom dian the frustum. Required proof.

Let A = the altitude of the cone.

a = the altitude of the frustum.

Then by simil. $\Delta s.$ $A : \Delta :: a : \Delta - \delta$.

$$\therefore \frac{A}{\Delta} = \frac{a}{\Delta - \delta}$$

Also $\{\Delta^2 - \delta\sqrt{\Delta\delta}\} \cdot \frac{\Delta\, a\, \pi}{3(\Delta - \delta)} =$ content of the lower ungula.

And $\frac{1}{3}\pi\, \text{A}\, \Delta^2 =$ content of whole cone.

$\therefore \frac{1}{3}\pi\, \text{A}\, \Delta^2 - \frac{1}{3}\pi\, \text{A}\, \{\Delta^2 - \delta\sqrt{\Delta\delta}\} = \frac{1}{3}\pi\, \text{A}\, \delta\sqrt{\Delta\delta} =$ the top part of the cone.

And therefore this top part : whole cone :: $\frac{1}{3}\pi\, \text{A}\, \delta\sqrt{\Delta\delta}$: $\frac{1}{3}\pi\, \text{A}\, \Delta^2$ :: $\delta^{\frac{3}{2}} : \Delta^{\frac{3}{2}}$. Q. E. D.

Cor. sq. of whole cone : sq. of top part : : $\Delta^3 : \delta^3$.

81. Having given the area of an hyperbola $= 4\,x\,y\,\left\{\frac{1}{3} - \frac{1}{1\cdot3\cdot5}\cdot\frac{x}{2\,a+x} - \frac{1}{3\cdot5\cdot7}\cdot\frac{x^2}{(2\,a+x)^2} - \frac{1}{5\cdot7\cdot9}\cdot\frac{x^3}{(2\,a+x)^3} - \text{\&c.}\right\}$ where x is the abscissa reckoning from the vertex, and y = corresponding ordinate; it is required to show that if the abscissa x be increased without limit, the area of the hyperbola becomes ultimately equal to the area of a triangle whose base is $2\,y$ and altitude x.

$$\text{Area} = 4\,x\,y\left\{\frac{1}{3} - \frac{1}{1\cdot3\cdot5}\cdot\frac{x}{2\,a+x} - \frac{1}{3\cdot5\cdot7}\cdot\frac{x^2}{(2\,a+x)^2} - \text{\&c.}\right\}$$

Let x become greater than any assignable magnitude, then $\frac{x}{2\,a+x} = \frac{x}{x} = 1$; and $\frac{x^2}{(2\,a+x)^2} = \frac{x^2}{x^2} = 1$, &c.

$$\therefore \text{Area} = 4\,x\,y\left\{\frac{1}{3} - \frac{1}{1\cdot3\cdot5} - \frac{1}{3\cdot5\cdot7} - \text{\&c. ad inf.}\right\}$$
$$= 4\,x\,y\left\{\frac{1}{3} - \left(\frac{1}{1\cdot3\cdot5} + \frac{1}{3\cdot5\cdot7} + \text{\&c. ad inf.}\right)\right\}$$

Now the sum of n terms of $\frac{1}{1\cdot3\cdot5} + \frac{1}{3\cdot5\cdot7} + \text{\&c.} =$
$\frac{1}{12} - \frac{1}{4\cdot(2\,n+1)\,(2\,n+3)}$

And this when n is infinitely great, becomes $= \frac{1}{12}$

$\therefore$ Area of the hyperbola $= 4\,x\,y\left\{\frac{1}{3} - \frac{1}{12}\right\} = x\,y = \frac{1}{2}\,x \times 2\,y =$ area of a rectilineal triangle whose altitude is x, and base $2\,y$.

82. Required the content of a tub shaped like the frus-

of a cone) supposing the top to be 60, the diagonal 66, and the length of a stave 30 inches.

If a straight line be drawn from the edge of the base perpendicular to the top diameter, we have 60 : 66 + 30 :: 66 — 30 : difference of segments of top diameter.

$$\therefore \text{diff. of those segments} = \frac{36 \times 96}{60} = 57{\cdot}6 = \text{bot. diam.}$$

and $\frac{1}{2}$ diff. of segments $= 28{\cdot}8$

$\frac{1}{2}$ sum of segments $= 30{\cdot}$

$\therefore$ lesser segment $= 1{\cdot}2$

And alt. of the tub $= \sqrt{30^2 - (1{\cdot}2)^2} = \sqrt{900 - 1{\cdot}44} = \sqrt{898{\cdot}56} = 29{\cdot}976$.

Hence content of the tub $= \dfrac{60^3 - (57{\cdot}6)^3}{60 - 57{\cdot}6} \times 9{\cdot}992 \times {\cdot}7854$ cubic inches.

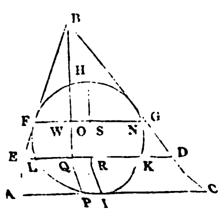
$$= \frac{24897{\cdot}024}{2{\cdot}4} \times 7{\cdot}8477168 = 24897{\cdot}024 \times 3{\cdot}26988.$$

$= 81411{\cdot}2808$ cubic inches $= 293{\cdot}61$ imperial gallons.

83. A circle is inscribed in a triangle whose three sides A B, A C, B C, are 13, 14, 15 feet respectively; and the triangle is divided into three equal parts by lines drawn parallel to A C; it is required to find the areas of the three portions of the inscribed circle.

$$\begin{array}{r} 13 \\ 14 \\ 15 \\ \hline \text{Perimeter} = 42 \\ \tfrac{1}{2}\text{ ditto} = 21 \end{array}$$

$\therefore$ area of triangle $= \sqrt{21 \times 8 \times 7 \times 6} = 84$ sq. feet.



And area also $= BP \cdot \frac{AC}{2} = 7\,BP \therefore BP = 12 =$ perpendicular altitude of the triangle. If the centre of the circle were found, and radii drawn to the points in which the sides of the triangle touch the circle, it will appear that area of the triangle $= \{AB + BC + AC\} \cdot \frac{\text{rad.}}{2} = 84$.

Or $42 \cdot \frac{\text{rad.}}{2} = 84 \therefore$ radius $= 4$, and the diameter of the inscribed circle $= 8$ feet.

Now the triangles BFG, BED, BAC, are similar, hence

$$\triangle BFG : \triangle BAC :: BO^2 :: BP^2$$
$$\text{Or } 1 : 3 :: BO^2 : 12^2$$
$$\therefore BO = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$
$$\text{And } \triangle BFG : \triangle BED :: BO^2 : BQ^2$$
$$\text{Or } 1 : 2 :: 48 : BQ^2 = 96$$
$$\therefore BQ = \sqrt{96} = 4\sqrt{6}$$
$$\text{Hence } BP - BQ = 12 - 4\sqrt{6} = PQ = RI.$$
$$\text{And } BP - BO = 12 - 4\sqrt{3} = OP = SI.$$
$$\text{And } IH - IS = 4\sqrt{3} - 4 = SH.$$

We have now found two versed sines HS, IR $= 4\sqrt{3} - 4$, $12 - 4\sqrt{6}$ respectively;

$$\left.\begin{array}{l}\text{And } \frac{4\sqrt{3}-4}{8} = \cdot 3660254 \\ \text{Also } \frac{12-4\sqrt{6}}{8} = \cdot 2752551\end{array}\right\} \begin{array}{l}\text{The corresponding} \\ \text{tab. vers. sines.}\end{array}$$

Tab. area corr. to ·3660254 is ·2603444
Tab. area corr. to ·2752551 is ·1757798

Sum = ·4361242
Area of circle to diameter 1 = ·7853981

Area of the zone (diam. being 1) = ·3492739

Hence $8^2 \times \cdot 2603444 = 16\cdot 662047 =$ area of seg. KIL.
$8^2 \times \cdot 1757798 = 11\cdot 249909 =$ area of seg. WHN.
$8^2 \times \cdot 3492739 = 22\cdot 353525 =$ area of zone WNKL

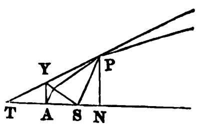
84. Two sides, A B, B C of a trapezium at right angles to one another are 6·5 and 15·6 feet respectively; also the other two sides are 12 and 9 : find the area of the trapezium.

$$AB^2 + BC^2 = \sqrt{(6{\cdot}5)^2 + (15{\cdot}6)^2} = 16{\cdot}9 = AC.$$

Hence we have two triangles; one right angled at B; and the other triangle an oblique angled one. The perimeter of the latter $= 37{\cdot}9$;

And its area $= \frac{1}{4}\sqrt{37{\cdot}9 \times 4{\cdot}1 \times 13{\cdot}9 \times 19{\cdot}9} = \frac{1}{4}\sqrt{42982{\cdot}4279}$ $= 51{\cdot}8305$ sq. feet. Also the area of the right angled triangle $= \frac{1}{2} \times 6{\cdot}5 \times 15{\cdot}6 = 50{\cdot}7$, consequently the area of the trapezium $= 102{\cdot}5305$.

85. If A be the vertex of a parabola, P any point in the curve, and S the focal distance; the area A S P $= a^2 \left\{ \frac{t^3}{3} + t \right\}$ where $a =$ S P, and $t =$ tangent of $\frac{1}{2}$ angle A S P.



Draw P T a tangent at P, also let P N and S Y be perpendicular to A N and P T respectively. Join A Y, which will be parallel to P N from the nature of the parabola.

Now S A : A Y :: 1 : t

$\therefore$ A Y $= a\,t$ and P N $= 2\,a\,t$

Also T A : A Y :: A Y : A S

Or T A : $a\,t$ :: $a\,t$: a :: t : 1

$\therefore$ T A or A N $= a\,t^2$

and S N $= a\,t^2 - a = a\,\{t^2 - 1\}$

The area $\text{ASP} = \text{ANP} - \text{SNP}$

$$= \tfrac{2}{3}\,\text{AN}\cdot\text{NP} - \tfrac{1}{2}\,\text{SN}\cdot\text{NP}$$
$$= \tfrac{2}{3}\,a\,t^2 \times 2\,a\,t - \tfrac{1}{2}\,a\,\{t^2 - 1\} \times 2\,a\,t$$
$$= \tfrac{4}{3}\,a^2\,t^3 - a^2\,t^3 + a^2\,t$$
$$= a^2\left\{\frac{t^3}{3} + t\right\}.$$

86. There is a piece of timber round and tapering like the frustum of a cone; its top and bottom diameters are 40 and 50 inches, and its altitude 6 feet: required the contents of the two hoofs into which it is divided by a plane passing through the extremity of the lesser diameter, and drawn parallel to the side of the timber.

In this question we have the frustum of a cone divided by a plane parallel to a side of the cone; hence the content of the

$$\text{lesser hoof} = \left\{\frac{\text{A}\,\Delta}{\Delta - \delta} - \frac{4\,\delta}{3}\,\{\delta\,(\Delta - \delta)\}^{\frac{1}{2}}\right\}\cdot\frac{a}{3}$$

Here $\Delta = 50$, $\delta = 40$, $a = 6$ feet $= 72$ inches.
Also A = circular area whose ver. sin. = 10 to diameter 50.
Now $\frac{10}{50} = \cdot 2$

And tab. area corr. to ·2 is ·111823

$$\begin{array}{rl}
 & 2500 = 50 \\ \hline
 & 55911500 \\
 & 223646 \\ \hline
\therefore \text{A} = & 279\cdot5575 \\
 & 50 = \Delta \\ \hline
\Delta - \delta = 10) & 13977\cdot8750 \\ \hline
 & 1397\cdot7875 = \frac{\text{A}\,\Delta}{\Delta - \delta}
\end{array}$$

$$\begin{array}{rl}
\text{Again } \Delta - \delta = & 10 \\
\delta = & 40 \\ \hline
\delta\,(\Delta - \delta = & 400
\end{array}$$

$$\therefore \{\delta(\Delta-\delta)\}^{\frac{1}{2}} = 20$$

$$53{\cdot}3333 = \frac{4\,\delta}{3}$$

$$\frac{4\,\delta}{3}\{\delta(\Delta-\delta)\}^{\frac{1}{2}}\ 1066.6660$$

$$1397{\cdot}7875$$

$$331{\cdot}1215$$

$$24 = \frac{a}{3}$$

$$13244860$$

$$6622430$$

7946·9160 cubic inches = content of lesser ungula. Also, content of the whole piece of timber $= \frac{\Delta^3-\delta^3}{\Delta-\delta}\cdot\frac{a}{3}\times{\cdot}7854 = \frac{50^3-40^3}{50-40}\cdot\frac{72}{3}\times{\cdot}7854$

$= (5^3-4^3)\times 100\times 24\times{\cdot}7854 = 166400\times{\cdot}7854.$

$$166400$$

$${\cdot}7854$$

$$665600$$

$$8320$$

$$13312$$

$$11648$$

$$130690{\cdot}5600$$

$$7946{\cdot}916$$

$122743{\cdot}644$ = solidity of the greater, or complemental hoof.

87. A tankard in the form of a cylinder whose diameter is 4 inches, and altitude 6 inches, being filled with negus worth 18*d*., is offered to A, who drings till he can see the centre of the tankard's base, and then gives the remainder to B. Find what part of the 18*d*. ought to be paid by A for the share of negus which he drank.

In this example we have to find the content of a cylindrical *ungula*, when the section passes through the centre of the base.

Now in general the content $= \{\frac{2}{3}$ PN^3— area of base × cos arc BP$\}. \frac{QB}{BN}$(See Case III. of cylindrical ungulas, in the Miscellaneous Problems given in the Appendix to the Mensuration.)

In this case the arc BP $= 90°$ ∴ its cosine $= 0$; also BN $=$ PN.

∴ the content $= \frac{2}{3}$ PN^2 × QB $= \frac{2}{3} \times \frac{AB^2}{4} \times$ QB $= \frac{1}{6}$ AB QB $=$ content of the liquor left by A. Also $\pi \cdot$ AB^2 QB $=$ content of the whole tankard; therefore

The whole : less share : : $\pi \cdot$ AB^2 QB : $\frac{1}{6}$ AB^2 QB : : ·7854 :
: : 4·7124 :

That is 18 pence : less share : : 4·7124 : 1.

Hence B must pay $\frac{18}{4·7124}$ pence, or $3\frac{3}{4}d.$; and A must pa $14\frac{1}{4}d.$

88. There is a punch-bowl in the form of the segment of prolate spheroid, whose axes are as 3 to 4; the depth of th bowl is one-fourth of the whole transverse axes, and the diamete of its top 20 inches. It is required to determine what numbe of rounds a company of 10 men may drink out of it, whe filled with liquor; the men using conical glasses, the depth an top diameter of each being 2 and $1\frac{1}{2}$ inches respectively.

Now if we take a spheroid whose axes are 3 and 4, and cu off from it a segment whose altitude is 1, this segment will b similar to the punch-bowl. And, by the nature of the ellipse

$$4 : 3 : : \sqrt{(4-1) \times 1} : \text{radius of the segment's base.}$$

$$\therefore \tfrac{3}{2}\sqrt{3} = \text{diameter of the segment's base.}$$

And the content of this segment $= \frac{3^2}{4^2}(12-2) \times 1^2 \times ·523$

$$= \tfrac{90}{16} \times ·5236 = 2·94525$$

Now similar solids are as the cubes of their like dimensions,

$$\therefore (\tfrac{3}{2}\sqrt{3})^3 : 20^3 : : 2·94525 : \text{content of the punch-bowl.}$$

$$\text{And the content} = \frac{20^3 \times 8}{81\sqrt{3}} \times 2·94525$$

$$\begin{aligned} 3 \text{ Lo. } 20 &= 3{\cdot}9030900 \\ \text{Log. } 8 &= {\cdot}9030900 \\ \text{Log. } 2{\cdot}94525 &= {\cdot}4691223 \\ \hline \text{Log. numerator} &= 5{\cdot}2753023 \\ \text{Log. } 81 &= \bar{1}{\cdot}9084850 \\ \tfrac{1}{2} \text{ Log. } 3 &= {\cdot}2385606 \\ \hline \text{Log. denom.} &= 2{\cdot}1470456 \end{aligned}$$

Log. content $= 1343{\cdot}55 \qquad 3{\cdot}1282567$

Also $\cdot 7854 \times (\frac{3}{2})^2 \times \frac{2}{3} = \cdot 7854 \times \frac{3}{2} =$ content of one glass,

And the quantity drunk at each round $= \cdot 7854 \times 15 = 11{\cdot}781$.

$\therefore$ the number of rounds required $= \dfrac{1343.55}{11{\cdot}781} = 114{\cdot}04$.

89. How many gallons, imperial measure, are contained in a cask composed of two equal frustums of a paraboloid; the length of the cask being 45 inches, the bung diameter 40, and the head diameter 20 inches?

Now the content of a parabolic frustum whose diameters are Δ, δ, and altitude a, $= \{\Delta^2 + \delta^2\}.\dfrac{\pi a}{8}$, where $\pi = 3{\cdot}14159$, &c.

In this example, $\Delta = 40$, $\delta = 20$, $a = 22\frac{1}{2}$

$\therefore$ content of one frustum $= \{40^2 + 20^2\} \times \cdot 3927 \times 22\frac{1}{2}$

And the content of the cask $= \{40^2 + 20^2\} \times \cdot 3927 \times 45$

$= 785{\cdot}4 \times 45 = 35343$ cubic inches.

And $\dfrac{35343}{277.274} = 127{\cdot}1$ imperial galls.

90. A party of thirty gentlemen sat down to drink a bowl of punch, and used glasses in shape of paraboloids; it is required to determine the number of bumpers each gentleman may drink, supposing the bowl to be a segment of an oblate spheroid whose axes are 50 and 30 inches respectively, and altitude 6 inches; also the altitude and top diameter of one of the glasses to be 3 and $2\frac{1}{2}$ inches respectively.

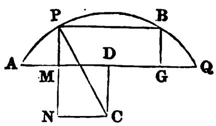
The content of the segment is found by Problem XIX. in the Mensuration. Here the spheroid is oblate, the fixed axis is 30 and the revolving axis 50, therefore

$$\text{The content} = \frac{50^2}{30^2}\{90 - 12\} \times 6^2 \times \cdot 5236$$
$$= \frac{25}{9} \times 78 \times 6^2 \times \cdot 5236$$
$$= 25 \times 26 \times 12 \times \cdot 5236$$
$$= 4084{\cdot}08 \text{ cubic inches.}$$

$$\text{And the content of a glass} = \cdot 7854 \times (2{\cdot}5)^2 \times \tfrac{3}{2}$$
$$= \cdot 7854 \times 6{\cdot}25 \times 1{\cdot}5$$
$$= 7{\cdot}36312 \text{ cubic inches.}$$

$$\therefore \frac{4084{\cdot}08}{220{\cdot}8936} = 18{\cdot}4 \text{ bumpers for each person.}$$

91. Find the area of the greatest rectangle that can be inscribed in the segment of a circle whose radius is 50 inches; the chord of the segment being 60 inches.



Let C be the centre of the given segment A P B Q whose chord is A Q. Suppose any point M be taken in A Q, and that P M is drawn perpendicular to A D, and being produced meets C N at right angles in N. Join C P; and complete the rectangle P B G M.

Let P M $= x$, C P $= r$, C D $= a$, and A D $= b$,

Then D M $=$ C N $= \{r^2 - (x + a)^2\}^{\frac{1}{2}}$

Now the area P M G B $=$ P M $\times$ G M and $\propto$ P M $\times$ M D, for M G $= 2$ M D.

$\therefore x \cdot \{r^2 - (x+a)^2\}^{\frac{1}{2}}$ is a maximum.

Or $x^2 \{r^2 - (x+a)^2\}$ is a maximum.

$\therefore 2\,x\,d\,x\,\{r^2 - (x+a)^2\} - 2\,x^2\,d\,x\,(x+a) = 0.$

Or $r^2 - (x+a)^2 - x\,(x+a) = 0$

i. e. $r^2 - 2\,x^2 - 3\,a\,x - a^2 = 0$

Whence $2\,x^2 + 3\,a\,x = r^2 - a^2$

and $x^2 + \frac{3\,a}{2} . x = \frac{r^2 - a^2}{2}$

$$\therefore x^2 + \frac{3\,a}{2} . x + \frac{9\,a^2}{16} = \frac{a^2 + 8\,r^2}{16} = \frac{9\,r^2 - b^2}{16}$$

$$\text{and } x = \frac{(9\,r^2 - b^2)^{\frac{1}{2}} - 3\,a}{4} = \text{PM}.$$

$$\text{Also } 2\,\text{CN} = 2\,\text{DM} = \text{MG} = 2\,\{r^2 - (x+a)^2\}^{\frac{1}{2}}$$

$$= 2\left\{r^2 - \left(\frac{(9\,r^2 - b^2)^{\frac{1}{2}} + a}{4}\right)^2\right\}^{\frac{1}{2}}$$

$$= 2\left\{r^2 - \frac{9\,r^2 - b^2 + 2a\sqrt{9\,r^2 - b^2} + a^2}{16}\right\}$$

$$= \left\{\frac{7\,r^2 + b^2 - 2\,a\sqrt{9\,r^2 - b^2} - a^2}{4}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{8\,r^2 - 2\,a\sqrt{9\,r^2 - b^2} - 2\,a^2}{4}\right\}^{\frac{1}{2}}$$

$$= \left\{2\,r^2 - \frac{a}{2}\left(\sqrt{9\,r^2 - b^2} + a\right)\right\}^{\frac{1}{2}}$$

$\therefore$ area of rectangle PMGB

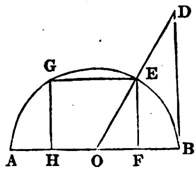
$$= \frac{\sqrt{9\,r^2 - b^2} - 3\,a}{4} . \{2\,r^2 - \frac{a}{2}(\sqrt{9\,r^2 - b^2} + a)\}^{\frac{1}{2}}$$

But $r = 50$, $b = 30$, $a = \sqrt{r^2 - b^2} = \sqrt{2500 - 900} = \sqrt{1600} = 40.$

$\sqrt{9\,r^2 - b^2} = \sqrt{22500 - 900} = \sqrt{21600} = 147$, very nearly.

$\therefore$ Area of the maximum rectangle $= \frac{27}{4}\{5000 - 20 \times 167\}^{\frac{1}{2}}$
$= \frac{27}{4}\sqrt{1660} = 275{\cdot}015$ square inches.

92. Inscribe a square in a given semicircle whose radius is 1 foot, and find the area of that square.



Let A G E B be the given semicircle, whose centre is O and diameter A B. Draw B D perpendicular and equal to A B. Join O D cutting the circle in E; then if E F be drawn perpendicular to A B, and E G, and G H respectively parallel to A B, E F, the figure E G H F will be a square.

For by similar triangles

$$\text{D B} : \text{B O} :: \text{E F} : \text{F O}$$

And by construction $\text{D B} = 2\ \text{B O}$,

$$\therefore \text{E F} = 2\ \text{F O} = \text{F H} = \text{E G}$$

Consequently the figure E G H F is equilateral; but E F H is a right angle, ∴ the figure is rectangular; and it has been proved to be equilateral, therefore it is a square.

Now $\text{O F} : \text{F E} :: 1 : 2$

$$\therefore \frac{\text{O F}^2}{\text{F E}^2} = \frac{1}{4}$$

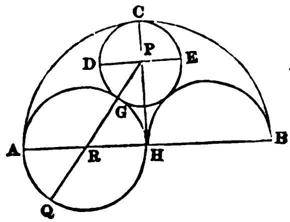
$$\text{Or } \frac{\text{O E}^2 - \text{E F}^2}{\text{E F}^2} = \frac{1}{4}$$

$$\therefore 4\ \text{O E}^2 = 5\ \text{E F}^2$$

And $\text{E F}^2 = \frac{4}{5}\ \text{O E}^2 = \frac{4}{5}$, since $\text{O E} = 1$,

i. e. area of the square $= \frac{4}{5} = \cdot 8$ of a square foot
$= 115 \cdot 2$ square inches.

93. On the two halves of the diameter of a semicircle let two equal circles be described, and in the curvilinear space let another circle be inscribed, it is required to determine the area of this last mentioned circle, supposing the diameter of the given semicircle to be 60 inches.



Let A B be the given diameter whose middle point is H. On A H, H B describe two circles, and in the remaining curvilinear space describe the circle C D E, touching the two former circles. From H draw H C perpendicular to A B, and H C will pass through the centre P of the circle C D G E. Take P and R the centres of the circles C D E, A Q H, then P Q will pass through the centre point of contact G. (Euclid B. 3. Prop. 12.)

Now Q P, P G = H P² Euclid B. 3. Prop. 37.

$\therefore$ PG : PH :: PH : PQ

And PG : PH :: PG + PH : PH + PQ

:: HC : PH + PG + GQ

:: HC : AC + HC

:: 1 : 2

$\therefore$ PG : CH :: 1 : 3

2 PG : CH :: 2 : 3

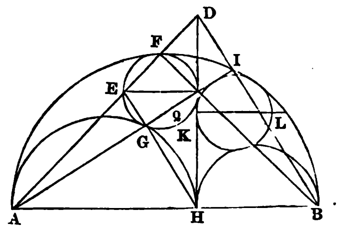
Or $2\ PG = \frac{2}{3} . CH = \frac{2}{3} \times 30 = 20$ inches = diameter of the circle C D G E.

$\cdot 7854$ = area of circle to diam. 1.

$400 = 20^2$

Area of circle C D G E = $314 \cdot 1600$ square inches.

94. Supposing the diameter of the semicircle to be 60 inches, as in the last problem, and to be divided in the point H into two parts which are to one another as 3 to 2; then if circles be described on these two parts, and a perpendicular drawn from H, it is required to show that the circles described in the curvilinear space, one on each side of the perpendicular, will be equal to each other; also determine the area of one of them.



Let A B be the given diameter whose length is 60 inches, and suppose A H : H B : : 3 : 2.

$\therefore$ A H $=$ 36 inches, and H B $=$ 24 inches.

On A B describe the semicircle A F I B, and on A H, H B, describe semicircles. From H draw H D perpendicular to A B, and on each side of H D let a circle be described touching the perpendicular and each of the semicircles A G H, A F B.

Suppose one of these circles to touch the perpendicular in Q, and the semicircles in F, G. Draw the diameter Q E parallel to A B, and join F E, E A, then F E A will be a straight line. Produce A F to meet the perpendicular in D. Join F Q, Q B, and F Q B will be a straight line, and the angle at F will be a right angle. Join E G, G H, Q G, G A, then will E H, Q A, be straight lines. Produce A Q to I, and join B I which will form the right angle B I A, also B I produced will pass through D; for since the perpendiculars from the angles upon the sides of a triangle meet in a point, therefore conversely, if the perpendiculars meet in a point as Q, the lines A B, B D, D A, will form a triangle. Now the right angle A G H $=$ A I B,

Therefore A D : D E : : A B : B H
Also A D : D E : : A H : E Q
$\therefore$ A B : B H : : A H : E Q

In a similar manner it may be proved that

A B : A H : : B H : K L
$\therefore$ E Q $=$ K L.

Now since AB : BH :: AH : EQ

$\therefore 60 : 24 :: 36 : EQ = 14{\cdot}4$ inches.

$$\begin{array}{r l} \cdot 7854 = & \text{area of a cir. to diam. 1.} \\ 207{\cdot}36 = & (14{\cdot}4)^2 \\ \hline 82944 & \\ 103680 & \\ 165888 & \\ 145152 & \\ \hline 162{\cdot}860544 = & \text{area of the circle FEGQ, or KL.} \end{array}$$

95. A wheel in 36 revolutions passes over 29 yards; and in x of these revolutions describes z yards, y feet, 5 inches: find the values of x, y, and z.

Since the whole passes over 29 yards in 36 revolutions, therefore it passes over $\frac{29}{36}$ yards in one revolution, and therefore passes over $\frac{29\,x}{36}$ yards in x revolutions; hence by the condition of the problem

$$\frac{29\,x}{36} = \overset{\text{yds.}}{z} + \overset{\text{ft.}}{y} + \overset{\text{in.}}{5}$$

$$= \overset{\text{yds.}}{z} + \overset{\text{yds.}}{\frac{y}{3}} + \overset{\text{yds.}}{\frac{5}{36}}$$

Or $29\,x = 36\,z + 12\,y + 5$

$$\therefore y = \frac{29\,x - 36\,z - 5}{12}$$

$$= 2\,x - 3\,z + \frac{5\,x - 5}{12}$$

And since y is supposed to be an integer, $\frac{5\,x - 5}{12}$ is an integer

And therefore $\frac{x-1}{12}$ is an integer.

Assume $\frac{x-1}{12} = p$.

$$\therefore x = 12p + 1$$

0, $x = 1$, but $\frac{x-1}{12}$ is not an integer as it ought to be.

1, $x = 13$, the least integral value of x.

Now $29x = 36z + 12y + 5$

Or $377 = 36z + 12y + 5$

$\therefore 36z + 12y = 372$

Or $3z + y = 31$

and $z = \frac{31-y}{3} = 10 + \frac{1-y}{3}$

$\therefore \frac{1-y}{3}$ is an integer.

Assume $\frac{1-y}{3} = q$

and $y = 1 - 3q$

$= 0$, $y = 1$, and $z = 10$; hence the least integral f x, y, and z, are 13, 1, 10. Another set of values ound to be 25, 3, 19, &c., &c.

if the wheel passes over 29 yards in 36 revolutions, it over 10 yds. + 1 ft. + 5 in. in 13 revolutions; and yds. + 3 ft. + 5 in. in 25 revolutions, &c. &c.

ere are two regular polygons, and one of them has nany sides as the other; also each of the interior angles st polygon is greater than each of the interior angles cond polygon in the ratio of 4 to 3; determine the f sides belonging to each figure.

= the number of sides belonging to one polygon.

d $\frac{x}{2}$ = the number of sides belonging to the other.

e all the interior angles of any rectililineal figure, h four right angles, are equal to twice as many the figure has sides. Euc. B. 1 Pr. 32. Cor. 1.

int. angles $+ 360° = 180x$

rior angle $= \frac{180x - 360}{x}$

the polygon having $\frac{x}{2}$ sides,

$$\frac{x}{2} \times \text{ int. angles } + 360° = 90\,x$$

$$\therefore \text{ one int. angle } = \frac{180\,x - 720}{x}$$

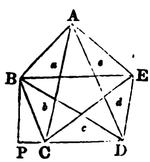
But the question says that the former interior angle is to the latter as 4 : 3;

$$\therefore \frac{180\,x - 360}{x} : \frac{180\,x - 720}{x} :: 4 : 3$$

$$\text{Or } 540\,x - 1080 = 720\,x - 2880$$

Whence $x = 10 =$ the number of sides belonging to the first polygon; and $\frac{x}{2} = 5 =$ the number of sides belonging to the second; therefore the two polygons are a decagon and a pentagon.

97. If the alternate angles of a given equilateral and equiangular pentagon be joined, there will be formed a small equilateral and equiangular pentagon. It is required to determine its area, supposing a side of the given pentagon be equal to 5 feet.



Let A B C D E be the given equilateral and equiangular pentagon. Join A C, B D, C E, D A, E B, which will cut one another in a, b, c, d, e, and form the figure $a\ b\ c\ d\ e$.

Now since the side A E = the side B C, the arc A E = arc B C (supposing a circle to be described round the pentagon), hence A B is parallel to C E; for the same reason A E is parallel to B D, and the figure A B c E is a parallelogram; whence A E = B c, and A B = E c, but A E = A B, ∴ B c = E c, and each is equal to a side of the pentagon. The same may be proved with regard to A b, B e, D e.

Again, since the angle D C c = the angle D B C = B D C, therefore the triangles D C c, D B C are similar: hence

$$\text{B D} : \text{D C} :: \text{D C} : \text{D C}$$

$$\text{Or B D} : \text{B C} :: \text{B C} : \text{D C},$$

that is, the point c divides B D in extreme and mean ratio. It appears from this, that $\text{BC} = b\ \text{D} = \text{C}\ d = c\ \text{E} = d\ \text{A} = \text{D}\ e = \&c.$

$\therefore b\,c = cd = d\,e = e\,a = a\,b$, and the figure $a\,b\,c\,d\,e$ is an equilateral and therefore an equiangular pentagon.

Draw B P perpendicular to D C produced in P.

$$\text{Then D B}^2 = \text{D C}^2 + \text{C B}^2 + 2\ \text{D C} \cdot \text{C P}$$

$$\text{but C B} : \text{C P} :: 1 : \cos.\ \text{B C P}$$

$$:: 1 : \cos.\ 72^\circ$$

$$\therefore \text{D B}^2 = \text{D C}^2 + \text{C B}^2 + 2\ \text{D C} \cdot \text{C B.}\ \ \text{Cos.}\ 72^\circ$$

$$= 5^2 + 5^2 + 50.\ \ \text{Cos.}\ 72^\circ$$

$$= 50\ \{1 + \cos.\ 72^\circ\} = \tfrac{25}{2} \cdot (3 + \sqrt{5})$$

$$\therefore \text{D B} = 5 \left\{\frac{3 + \sqrt{5}}{2}\right\}^{\frac{1}{2}}$$

If $x = \text{D C}$

$$\text{B D} \cdot x = \text{B C}^2$$

$$\text{Or } 5\ x \left\{\frac{3 + \sqrt{5}}{2}\right\}^{\frac{1}{2}} = 25$$

$$\therefore x = 5 \cdot \left(\frac{2}{3 + \sqrt{5}}\right)^{\frac{1}{2}}$$

$$\text{and } b\,c = b\ \text{D} - \text{D C}$$

$$= 5\left\{1 - \left(\frac{2}{3 + \sqrt{5}}\right)^{\frac{1}{2}}\right\} = 5\{1 - \cdot618\} = 5 \times \cdot382$$

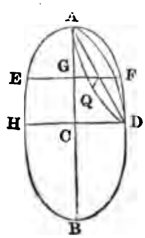
$$\therefore \text{area of given pentagon} : \text{area of } a\,b\,c\,d\,e :: 5^2 : 5^2 \times (\cdot382)^2$$

$$:: 1 : \cdot145924$$

$$\text{Or } 25 \times 1\cdot720477 : \text{area of } a\,b\,c\,d\,e :: 1 : \cdot145924$$

$$\therefore \text{area of pentagon } a\,b\,c\,d\,e = 6.2764 \text{ square feet.}$$

98. It is required to determine the area of the section of any spheroid, formed by a plane passing through the extremities of the two axes, whose lengths are 80 and 60 inches respectively.



Let A B, H D be the major and minor axes of the spheroid, which we will first suppose to be prolate. And let A Q D be the section passing through the extremities of the axes. Join A D which will be the major axis of the section A Q D. Through P the middle point in A D draw E P F perpendicular to A B.

$$\text{Then } AD = (AC^2 + CD^2)^{\frac{1}{2}} = \sqrt{2500} = 50.$$

$$\text{Also } BG \cdot GA : GF^2 :: AC^2 :: CD^2$$

$$\text{or } AC^2 - CG^2 : GF^2 :: AC^2 : CD^2$$

$$\therefore GF^2 = \frac{CD^2}{AC^2} \times (AC^2 - CG^2)$$

$$= \frac{CD^2}{AC^2} \times (AC^2 - \tfrac{1}{4}AC^2) = \tfrac{3}{4} CD^2.$$

Also since the solid is generated round the major axis A B, any section perpendicular to A B will be a circle. Let E F be the diameter of a section perpendicular to A B, and cutting the ellipse A Q D in Q, then 2 P Q will be the minor axis of the section A Q D.

$$\text{Now } 2PQ = 2\sqrt{EP \cdot PF}$$

$$= 2\sqrt{GF^2 - GP^2}$$

$$= 2\left(\tfrac{3}{4}CD^2 - \tfrac{1}{4}CD^2\right)^{\frac{1}{2}} = CD\sqrt{2}$$

$$\text{Hence } AD \times 2QP \times \cdot 7854 = 50 \times 30\sqrt{2} \times \cdot 7854$$

$$= 1500 \times \cdot 7854 \times \sqrt{2}$$

$$= 15 \times 78 \cdot 54 \times 1 \cdot 41421$$

$$= 1178 \cdot 1 \times 1 \cdot 41421$$

$$= 1666 \cdot 08 \text{ square inches the}$$

area of the section E Q D; the given spheroid being prolate, or generated round the major axis.

If the spheroid be oblate, or generated round the minor axis, it will be found that the minor axis of the section A Q D is equal to $AC\sqrt{2} = 40\sqrt{2}$, therefore the area

$$= AD \times 2PQ \times \cdot 7854$$

$$= 50 \times 40\sqrt{2} \times \cdot 7854$$

$$= 2000 \times \cdot 7854\sqrt{2}$$

$$= 1570.8 \times 1 \cdot 41421$$

$$= 2221 \cdot 441 \text{ square inches.}$$

99. The axes of a prolate spheroid are 50 and 40 inches; find the number of square feet in its surface. Also, the axes being the same, find the surface of the oblate spheroid.

In the Appendix to the Mensuration, it is shown at p. 325, that the surface $= 4\,\pi\, a\, b \left\{1 - \frac{\text{E}^2}{3}\right\}^{\frac{1}{2}}$ nearly; where $a =$ semi-major axis, $b =$ semi-minor axis, $\text{E}^2 = \frac{a^2 - b^2}{a^2}$ and $\pi = 3{\cdot}1416$.

Here $a = 25,\ b = 20,\ \text{E}^2 = \frac{25^2 - 20^2}{25^2} = \frac{225}{625} = \frac{9}{25}$;

$$\therefore \left(1 - \frac{\text{E}^2}{3}\right)^{\frac{1}{2}} = \left(1 - \frac{3}{25}\right)^{\frac{1}{2}} = \left(\frac{22}{25}\right)^{\frac{1}{2}} = {\cdot}93808.$$

$$\begin{aligned}\therefore \text{Surface required} &= 50 \times 40 \times 3{\cdot}1416 \times .93808 \\ &= 2000 \times 3.1416 \times {\cdot}93809 \\ &= 6283{\cdot}2 \times {\cdot}93809 = 5894{\cdot}207088 \text{ sq. inches.}\end{aligned}$$

With regard to the oblate spheroid, the surface $= 4\,\pi\, a\, b \left\{1 + \frac{\text{E}^2}{3}\right\}^{\frac{1}{2}}$ nearly, where $\text{E}^2 = \frac{a^2 - b^2}{b^2}$; which by substitution becomes $50 \times 40 \times 3{\cdot}1416\sqrt{1 + \frac{3}{16}} = 6283{\cdot}2 \times \sqrt{\frac{19}{16}} = 6283{\cdot}2 \times 1{\cdot}089725 = 6846{\cdot}96012$ square inches.

NOTE.—The expressions $4\,\pi\, a\, b \left\{\begin{matrix}1 - \\ + \end{matrix}\frac{\text{E}^2}{3}\right\}^{\frac{1}{2}}$ are merely approximations to the value of the series which express the true surfaces; and extend to two terms only of such series; hence the approximations are not very accurate.

The following expressions will give the surfaces more nearly than those which we have adopted above: but since they are rather tedious in their application, and in practice it rarely happening that great accuracy is required, the expressions will be merely mentioned.

For the prolate spheroid.

$$\text{The surface} = \frac{27}{7} \times 4\,\pi\, a\, b \left\{ \left(1 - \frac{\text{E}^2}{3}\right) - \frac{4}{9}\left(1 - \frac{\text{E}^2}{6}\right) - \frac{8}{27}\left(1 - \frac{\text{E}^4}{6} - \frac{\text{E}^2}{40}\right) \right\}$$

For the oblate spheroid,

The surface $= \frac{27}{7} \times 4 \pi a b \left\{ \left(1 + \frac{E^2}{3}\right)^{\frac{1}{2}} - \frac{4}{9}\left(1 + \frac{E^2}{6}\right) \right.$

$\left. \frac{8}{27}\left(1 + \frac{E^2}{6} - \frac{E^4}{40}\right) \right\}.$

In the former expression $E^2 = \frac{a^2 - b^2}{a^2}$; and in the lat

$E^2 = \frac{a^2 - b^2}{b^2}$.

If these expressions were used in the solution of the prob we should find the surface of the prolate spheroid $\doteq$ 5882·8 nearly, and the surface of the oblate spheroid = 6831· nearly.

THE END.

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